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A shell finite element based on a multiscale approach for out-of-plane analysis masonry walls

Daniela Addessi[1], Paolo Di Re[1], Elio Sacco[2]
[1] *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy*
E-mail:paolo.dire@uniroma1.it,daniela.addessi@uniroma1.it

[2] *Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*
E-mail:elio.sacco@unina.it

Keywords: Masonry, multiscale model, shell finite element

Masonry is a complex heterogeneous composite material made of bricks and/or blocks bonded together by cementitious mortar layers. As mortar is usually weaker than bricks and blocks and debonding phenomena usually develop at interfaces, numerical simulation of the nonlinear masonry behavior results a hard, yet important, task. Under severe loading conditions, such as seismic actions, if crumbling is prevented, out-of-plane failure mechanisms of masonry walls and vaults frequently occur, representing one of the main causes of structure collapse.

Discrete element methods, discontinuous deformation analysis and finite element (FE) models are the most used methods for numerical analysis of masonry structures. Among them, FE models based on multiscale approaches are often preferred, as these are based on an accurate geometric/mechanical representation of the material behavior with a limited computational cost. In these approaches, the response of the continuous medium at the higher geometric scale (macroscale) is determined by evaluating the response of a Unit Cell (UC) at a lower geometric scale (microscale) and applying specific homogenization procedures [1, 2]. Multiscale approaches applied to shell-like FE models have been proposed [3] to reproduce out-of-plane failure of masonry walls with significant computational advantages with respect to more complex three-dimensional formulations.

This work presents a thick shell FE for nonlinear analysis of periodic masonry walls under out-of-plane loads. A two-scale shell-to-solid approach is introduced linking the general quadrature point of the shell element at the macroscale with a three-dimensional UC at the microscale. Here, the displacement field is assumed as composed by an assigned part, directly related to the macroscopic shell strains, and an unknown perturbation, due to heterogeneous nature of masonry. Bricks constituting masonry walls are considered as linear elastic, while the mechanical behavior of mortar joints is modeled through a damage-friction constitutive law.

The proposed FE model is implemented in a standard FE code and is used to perform numerical simulations of masonry specimens subjected to out-of-plane loading conditions.

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Influence of Meso-Structure on the Mechanical Response of FDM 3D Printed Material

Gianluca Alaimo[1], Ferdinando Auricchio[1], Sonia Marfia[2], Elio Sacco[3]
[1]*Department of Civil Engineering and Architecture, University of Pavia, Italy*
E-mail: *gianluca.alaimo01@universitadipavia.it, auricchi@unipv.it*

[2]*Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, Italy*
E-mail: *marfia@unicas.it*

[3]*Department of Structures for Engineering and Architecture, University of Naples Federico II, Italy*
E-mail: *elio.sacco@unina.it*

Keywords: 3D Printing, Fused Deposition Modeling, Multiscale Approach.

Lately, Additive Manufacturing (AM) processes via 3D printers have made several progress with the aim of becoming rapid manufacturing methods to produce finished components. A crucial point in the use of AM solutions for direct production is the satisfactory evaluation of the mechanical properties of the printed components, so that they can be correctly accounted for in the design phase [1].

The present study focuses on the determination of the mechanical properties of Fused Deposition Modeling (FDM) 3D-printed objects. In FDM processes a thermoplastic filament is heated and extruded: the material is deposited layer by layer. Each printed layer is made of filaments, called fibers and deposited in a plane parallel to the printing surface, and of voids, since the deposited material is not able to full the space due to geometrical and process constraints. The obtained material can be considered as a composite with two phases, i.e. fibers and voids.

Some studies have been proposed in literature to study the response of FDM 3D-printed materials [2]. Accordingly, the aim of the present study is to develop a meso-macro analysis for studying the mechanical response of FDM 3D-printed material. A representative volume element (RVE) of the heterogeneous material made by the fibers and the voids is analyzed. Initially, a micromechanical approach based on nonlinear finite element analyses is developed. Finite elements characterized by 3D displacement fields that do not vary along the fiber axis are proposed. Then, a homogenization technique, based on Transformation Field Analysis [3], is implemented, introducing an approximation for the inelastic strain field. A proper nonlinear constitutive model for the filament is proposed. In order to assess the efficiency of the proposed micromechanical and homogenization approaches, comparisons with experimental data are performed.

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Multigrid algorithms for p–version Virtual Element methods

Paola F. Antonietti[1], Lorenzo Mascotto[2], Marco Verani[1]
[1] *MOX, Dipartimento di Matematica, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133, Milano, Italy*

E-mail:paola.antonietti@apolimi.it,marco.verani@polimi.it

[2] *Fakultat fur Mathematik, Universitat Wien, Oskar-Morgenstern-Platz 1, 1090 Wien, Austria*
E-mail:lorenzo.mascotto@univie.ac.at

Keywords: Multigrid methods, Virtual Elements, high-order

We present a multigrid algorithm for the solution of the linear systems of equations stemming from the numerical approximation of a second-order elliptic problem based on employing the p-version Virtual Element method [2, 3]. The construction of the multilevel method makes use of suitable interspace operators between a sequence of non-nested high-order Virtual Element spaces [1]. The convergence analysis of the resulting algorithm is based on the construction of suitable *auxiliary* local virtual spaces. Numerical experiments are shown which underpin the theoretical predictions; moreover, the proposed multilevel solver is shown to be convergent in practice, even when some of the theoretical assumptions are not fully satisfied.

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Curvilinear polygonal virtual elements for asymptotic homogenization problems

E. Artioli[1], L. Beirão da Veiga[2], C. Lovadina[3], M. Verani[4]

[1] *Dipartimento di Ingegneria Civile e Ingegneria Informatica, Università di Roma Tor Vergata, Roma, Italia E-mail:artioli@ing.uniroma2.it*
[2] *Dipartimento di Matematica e Applicazioni, Università di Milano Bicocca, Milano, Italia E-mail:lourenco.beirao@unimib.it*
[3] *Dipartimento di Matematica, Universitá di Milano, Milano, Italia E-mail:carlo.lovadina@unimi.it*
[4] *MOX, Dipartimento di Matematica, Politecnico di Milano, Milano, Italia E-mail:marco.verani@polimi.it*

Keywords: VEM, asymptotic homogenization, adaptive refinement, a posteriori error estimator

We present a virtual element method [1, 3] with curvilinear polygons for solving the unit cell problem, in application of the asymptotic homogenization method, and computing the antiplane shear homogenized material moduli of a composite material reinforced by cylindrical inclusions. Validation of the proposed numerical method is proved by comparison with analytical and numerical reference solutions, for a number of micro-structural arrays and for different properties of the material constituents. The presented study will focus on a posteriori error estimation [2] aiming at enhancing numerical efficiency by making use of local mesh refinement. The flexibility of the method allows for a large variety of microstructure shapes.

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An equilibrium-based stress recovery procedure for VEM

Edoardo Artioli[1], Stefano de Miranda[2], Carlo Lovadina[3,4], Luca Patruno[2]
[1] *Department of Civil Engineering and Computer Science, University of Rome Tor Vergata, Via del Politecnico 1, 00133 Rome, Italy E-mail:rtldrd00@uniroma2.it*

[2] *DICAM, University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy E-mail:stefano.demiranda@unibo.it, luca.patruno@unibo.it*

[3]*Dipartimento di Matematica, Università degli Studi di Milano, Via Saldini 50, 20133 Milano, Italy E-mail:carlo.lovadina@unimi.it*

[4]*IMATI del CNR, Via Ferrata 1, 27100 Pavia, Italy*

Keywords: VEM, stress recovery, RCP

A significant problem often arising within the framework of the displacement-based Virtual Element Method (VEM) for plane elasticity [1, 2], is represented by an accurate evaluation of the stress field. In particular, an operator which links the element degrees of freedom to the parameters governing the assumed strain field is introduced in the classical VEM formulation in order to allow the calculation of the stiffness matrix. The stress field is then computed using that strain field, by using the constitutive law. If a first-order formulation for a homogeneous material is assumed, for example, strains are locally mapped onto constant functions, and stresses are piecewise constant, accordingly. It must be nevertheless noticed that the virtual displacements might engender more complex strain fields for polygons which are not triangles.

In this contribution, Recovery by Compatibility in Patches (RCP) [3] is adapted to the VEM framework and used to devise an alternative stress recovery procedure, which allows to partly alleviate the above drawback and compute accurate stresses in the displacement-based VEM approach. The key idea underlying RCP is to treat patches of elements as isolated systems with known boundary displacements and minimise the associated complementary energy. On each patch, stresses are assumed to be a linear combination of self-equilibrated stress modes enriched by an appropriate particular solution. As the explicit knowledge of the displacements is required only along the patch boundaries, the RCP approach is naturally well-suited for virtual element schemes.

In particular, two approaches are considered: a degenerate patch composed of a single polygon is considered in the first one while, in the second one, RCP is adopted by considering patches of elements. This second approach indeed corresponds to its original formulation developed in the context of finite elements schemes. Numerical tests confirm the soundness of the proposed approach.

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A Virtual Element Method approach for 2D fracture mechanics problems

Edoardo Artioli[1], Sonia Marfia[2], Elio Sacco[3]
[1]*Department of Civil Engineering and Computer Science, University of Rome Tor Vergata, Rome, Italy*
E-mail: *artioli@ing.uniroma2.it*

[2]*Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, Italy*
E-mail: *marfia@unicas.it*

[3]*Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*
E-mail: *elio.sacco@unina.it*

Keywords: Virtual Element Method, Fracture, Element splitting.

The object of the present work is the development of an innovative methodology for 2D fracture mechanics problems based on the Virtual Element Method (VEM).
The VEM is a new technology for the approximation of partial differential equations, which can be interpreted as an evolution of modern mimetic finite difference schemes, and which shares the same variational background of the finite element method [1,2]. The main feature of the VEM is the possibility to construct an accurate Galerkin scheme with the flexibility to deal with highly general polygonal/polyhedral meshes, including “hanging vertices” and non-convex shapes, retaining the conformity of the method, i.e. the property to build an approximated solution which shares the same regularity features as the analytical solution of the problem under consideration. In many interesting cases, this means that the discrete solution is continuous across adjacent elements. Owing to the powerful features of the VEM in relation with mesh generation and enhanced topological tools, the analysis will point to assessing accuracy and efficiency in the aforementioned computational fracture mechanics problem presenting a comparison with more established techniques [3]. In particular, the analysis will point out some interesting issues made possible using the innovative method, which are more cumbersome when resorting to standard FEM, namely the easiness of fracturing and segmenting existing elements whenever a fracturing path crosses them.
Based on a wide variety of material models and fracture domain setups, an extensive numerical campaign will show the efficiency and accuracy of the proposed methodology which can then be seen as a powerful alternative classical method.

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The Virtual Element Method with curved edges

L. Beirão da Veiga[1], A. Russo[1], G. Vacca[1]
[1] *Department of Mathematics and Applications, University of Milano-Bicocca, Milano, Italy*
E-mail:lourenco.beirao@unimib.it, alessandro.russo@unimib.it, giuseppe.vacca@unimib.it

Keywords: Finite Element Method, Virtual Element Method, Curved edges

The Virtual Element Method (VEM) was introduced in [1, 2] as a generalization of the Finite Element Method that allows for general polygonal and polyhedral meshes. Polytopal meshes can be very useful for a wide range of reasons, including meshing of the domain (such as cracks) and data (such as inclusions) features, automatic use of hanging nodes, moving meshes, adaptivity. By avoiding the explicit construction of the local basis functions, Virtual Elements can easily handle general polygons/polyhedrons without the need of an overly complex construction.

The scope of the present talk is to present a first study [3] (on a simple model elliptic problem in 2D) of Virtual Elements with curved edges. Indeed, all the VEM papers in the literature make use of polygonal and polyhedral meshes, i.e. with straight edges and faces. On the other hand, as recognized in the finite element (FEM) literature, expecially for high order methods the approximation of the domain by facets introduces an error that can dominate the analysis. This issue has led, for example, to the development of non affine isoparametric FEM elements and to Isogeometric Analysis. In the context of Virtual Elements, one can exploit the peculiar construction of the method that i) does not need an explicit expression of the basis functions and ii) is directly defined in physical space, i.e. no reference element is used. This allows to define discrete spaces also on elements that are curved in such a way to exactly represent the domain of interest. The needed ingredient is a (piecewise regular) parametrization of the boundary of the domain. It is interesting to note that, although the initial construction and the theoretical proofs are clearly more involved, at the practical level the coding of the method with curved edges turns out to be essentially the same as in the straight edge case (the only difference being in the integration on edges and elements). Somehow, the present case fits very naturally into the Virtual Element setting.

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Virtual Elements for the Navier-Stokes equation with application to a leaflet problem

Lourenco Beirao da Veiga[1], Carlo Lovadina[2], Giuseppe Vacca[1]
[1] *Dipartimento di Matematica e Applicazioni, Università di Milano-Bicocca, Italy*
E-mail:lourenco.beirao@unimib.it,giuseppe.vacca@unimib.it

[2] *Dipartimento di Matematica, Università di Milano, Italy*
E-mail:carlo.lovadina@unimi.it

Keywords: Navier-Stokes, polygonal mesh, virtual elements

The Virtual Element Method (in short VEM, introduced in [1, 2]) is recent generalization of the Finite Element Method that enjoys also a connection with modern Mimetic schemes. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrix, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. For instance, the VEM easily allows for polygonal/polyhedral meshes (even non-conforming) also with non-convex elements and still yields a conforming solution with (possibly) high order accuracy; furthermore, it allows for discrete solutions of arbitrary C^k regularity, defined on unstructured meshes.

In the present talk we introduce the Virtual Element Method in the framework of fluid dynamics, more specifically the Stokes [3] and Navier-Stokes [4] equations. We present a method of general order of accuracy that (in addition to enjoying the important advantage of handling general polytopal meshes) exploits the flexibility of Virtual Elements in order to obtain an exactly divergence-free solution. This is well known to yield a set of advantages, when compared to more traditional inf-sup stable methods, that are explored both theoretically and numerically in [4]. After an introduction to the method, we present an application to a simple flow problem with a rigid leaflet where the VEM flexibility in terms of mesh generation is exploited.

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Flow and transport Virtual Element simulations in complex porofractured media

S. Berrone[1], A. Borio[1], A. D'Auria[1], S. Pieraccini[2], S. Scialò[1], F. Vicini[1]
[1] *Dipartimento di Scienze Matematiche, Politecnico di Torino, Torino, Italy*
[2] *Dipartimento di Ingegneria Meccanica e Aerospaziale, Politecnico di Torino, Torino, Italy*
E-mail:{*stefano.berrone, andrea.borio, alessandro.dauria, sandra.pieraccini, stefano.scialo, fabio.vicini*}*@polito.it*

Keywords: Discrete Fracture Network, Virtual Element Method, Flow and Transport

In many engineering problems the geometrical complexity of the computational domain is a key issue. Among such problems, flow simulation in poro-fractured media is particularly challenging as it is involving very complex domains that can generate several kinds of geometrical complexities, yielding in particular mesh generation problems. Subsurface flow simulations in fractured media are commonly required in the analysis of enhanced Oil&Gas simulations, CO_2 geological storage, water resources management and preservation, nuclear waste geological storage and high entalpy geothermal applications.

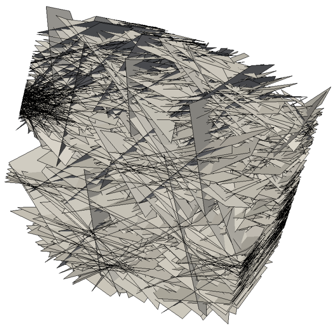

Figure 1: DFN example

In Discrete Fracture Network (DFN) models the fractures crossing the rock matrix are represented by polygons, and the flow equation is reduced to a flow in a planar domain, with suitable matching conditions coupling the flow at fracture intersections. Since the domains are usually stochastically generated, numerical methods should display robustness with respect to arbitrary geometrical complexities, jointly with efficiency, in such a way that a possibly very large number of simulations may be performed, for example, in case of uncertainty quantification techniques.

Virtual Element Methods can be fully exploited in order to avoid mesh generation problems. The VEM can be used for flow and transport simulations in the fractures neglecting the exchange with the surrounding rock matrix [1, 2, 3]; recently it has also been applied for flow simulations coupling the 2D flow in the fractures with the 3D flow in the surrounding rock matrix. Simulations in fractures highlighted the relevance of developing *ad hoc* strategies in order to tackle problems originated by elements with a huge aspect ratio that can be generated by the mesh generation process [4].

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Robustness of the Stabilisation Term in the Virtual Element Method

Silvia Bertoluzza[1], Micol Pennacchio[1], Daniele Prada[2]
[1] *Istituto di Matematica Applicata e Tecnologie Informatiche, CNR, Pavia, Italy*
E-mail:{*silvia.bertoluzza,micol.pennacchio,daniele.prada*}*@imati.cnr.it*

Keywords: *Virtual element method, stability.*

The design of the Virtual Element Method ([1]) relies on the choice of the degrees of freedom and on the form chosen for the stabilisation term which, acting directly on the degrees of freedom, aims at ensuring the coercivity of the approximate bilinear form by controlling the non-polynomial part of the discrete functions. Different options have been proposed (see, for instance, [2, 3]), and, in term of accuracy of the numerical solution, even quite simple design for such term (e.g. correct scaling of the degrees of freedom and diagonal stabilisation term) turn out to produce satisfactory results. However, understanding the impact that the design of such term has on the overall performance of method, by affecting quantities such as the condition number of the stiffness matrix, and how this depends on the quality of the tessellation and on the order of the method, is a crucial step towards designing robust $h-p$ VEM formulations on general tessellations. Such an understanding can be achieved by combining theoretical and experimental analysis and will hopefully lead to the design of reasonably cheap and nearly optimal choice of degrees of freedom and corresponding stabilisation term.

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Inelastic analysis of framed structures with generic cross-sections

Antonio Bilotta, Giovanni Garcea
Department of Informatics, Modeling, Electronics and System Engineering (DIMES), University of Calabria, Rende (CS), Italy
E-mail: antonio.bilotta@unical.it, giovanni.garcea@unical.it

Keywords: Iesan analysis, beam finite element, plasticity

The analysis of structural problems on the basis of one-dimensional models plays a central role in engineering applications. However standard approaches often are based on an over-simplified description of the cross-section behaviour and for this reason several new approaches, see for example [1, 2, 3], have been proposed with the intent to maintain the one-dimensional point of view but tying it to a richer description of the cross-section behaviour or, in general, to the actual 3D behaviour.

In the present work the semi-analytic FEM method already presented in [4, 5] for applications in the elastic field is used to devise a two-level computational framework for the inelastic analysis of framed structures. At the *frame level* the structure is described on the basis of a mixed beam element which uses only the displacement parameters of the end nodes, twelve in total, and six generalized stress parameters. The evaluation of the response of the beam element, in terms of generalized stress parameters and tangent stiffness matrix, is however tied to the *cross-section level* of the analysis. At this level the semi-analytic FEM approach is used to formulate a nonlinear problem over the cross-sections located in the control points and in the integration points of the beam element. The material behaviour, described in a full 3D context, is defined on the basis of a first-order elastic-plastic model, but the approach is enough general and the extension to other kind of material behaviours is possible.

The numerical experimentation performed with respect to several kinds of cross-sections confirms the effectiveness of the proposal which fits well a parallel implementation into multithreaded or High Performance Computing (HPC) environments.

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Formulazione di un modello di delaminazione in modo misto in presenza di fiber-bridging e transizione da piccole a grandi aperture

Federica Confalonieri[1], Umberto Perego[2]

[1]*DICAr, University of Pavia, Pavia, Italy*
E-mail: *federica.confalonieri@unipv.it*

[2] *DICA, Politecnico di Milano, Milano, Italy*
E-mail: *umberto.perego@polimi.it*

Keywords: Delaminazione, fiber-bridging, modello coesivo

Sebbene l'approccio coesivo fornisca uno strumento efficace per la simulazione agli elementi finiti di fenomeni di delaminazione, i modelli coesivi classici, formulati nell'ipotesi di piccole aperture, cadono in difetto in presenza di grandi aperture o di fiber-bridging. Nel primo caso, infatti, può verificarsi una violazione dell'equilibrio alla rotazione [1], mentre nel secondo lo sviluppo estensivo di fenomeni di fiber-brigding può causare un sensibile incremento dell'energia di frattura, tipicamente espresso in termini di curva R [2]. Inoltre, la delaminazione si sviluppa, in generale, in modo misto e la risposta meccanica è governata dalla presenza concomitante di sforzi normali e tangenziali e dall'interazione tra essi. In questo lavoro, il modello coesivo per la simulazione di fenomeni di delaminazione in modo misto sotto l'ipotesi di piccole aperture, descritto in [3], viene esteso per tener conto di fenomeni di fiber-bridging e del raggiungimento di grandi aperture. In regime di piccole aperture, si ricorre ad elementi di interfaccia classici: come proposto in letteratura [2], in puro modo I si adotta una legge sforzo coesivo-spostamento relativo di tipo trilineare, mentre in puro Modo II viene utilizzata una legge di tipo bilineare, coerentemente con il fatto che il fiber-bridging è governato da condizioni di carico di prevalente modo I. In presenza di grandi aperture, si propone invece l'utilizzo di un elemento di interfaccia a fibrilla, concettualmente simile all'elemento coesivo direzionale descritto in [4], che garantisce l'equilibrio alla rotazione per effetto della co-linearità di sforzi coesivi e spostamenti relativi. Viene inoltre introdotta una procedura di transizione da piccole a grandi aperture basata su un criterio di attivazione a danno crescente con il rapporto di modo misto, tale da evitare discontinuità nell'energia dissipata.

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Closed form solution of the return mapping algorithm for three-dimensional elastoplasticity

Fabio De Angelis[1]
[1]*Department of Structures for Engineering and Architecture,*
University of Naples Federico II, Naples 80125, Italy
E-mail: *fabio.deangelis@unina.it*

Keywords: Elastoplasticity, Constitutive problems, Nonlinear kinematic hardening.

In the present analysis small strain three-dimensional elastoplasticity boundary value problems are discussed with an exact closed form solution of the local constitutive equations, see De Angelis and Taylor [1][2]. Nonlinear kinematic hardening rules are considered in modelling the kinematic hardening behaviour of ductile materials [3]. In the literature a notable proposal for exact closed form solution of elastoplasticity problems has been presented e.g. by Simo and Govindjee [4] where the analysis is restricted to plane stress elastoplasticity and linear kinematic hardening. Conversely the present approach has the advantage to hold for three-dimensional elastoplasticity problems with nonlinear kinematic hardening rules. The present algorithmic procedure reduces the local solution of the constitutive equations to only one nonlinear scalar equation. Furthermore, in the present study a particularly simple form of nonlinear scalar equation is derived. In fact, herein the local constitutive equations are reduced to a single variable algebraic (polynomial) equation. Moreover, in the present approach due to the straightforward form of the nonlinear scalar equation the analytical solution of the algebraic equation can be found in exact closed form. Accordingly, a remarkable feature of the present approach is that no iterative solution method is used to solve the local constitutive equations in three-dimensional elastoplasticity. The expression of the consistent tangent operator associated to the proposed algorithmic scheme is also derived, thus ensuring a quadratic rate of asymptotic convergence when used with the Newton Raphson iterative method for the global solution procedure of the structural problem, see e.g. [5][6]. Numerical applications and computational results are finally reported for different types of complex and cyclic loading conditions. Accordingly, the robustness and effectiveness of the proposed algorithmic procedure is illustrated with specific numerical examples.

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A low-order virtual element formulation for the analysis and simulation of strain softening in solids

Maria Laura De Bellis[1], Peter Wriggers[2], Blaž Hudobivnic[2], Giorgio Zavarise[3]
[1] *Dipartimento di Ingegneria dell'Innovazione, Università del Salento, Lecce, Italy*
E-mail:marialaura.debellis@unisalento.it
[2] *Institute for Continuum Mechanics, Leibniz Universität Hannover, Germany*
E-mail:wriggers@ikm.uni-hannover.de,hudobivnic@ikm.uni-hannover.de
[3] *Dipartimento di Ingegneria Strutturale, Edile e Geotecnica, Politecnico di Torino, Italy*
E-mail:giorgio.zavarise@polito.it
Keywords: VEM, Stabilization, Isotropic damage, Regularization techniques

In this work we present a first order virtual element approach for modelling the strain-softening response of quasi-brittle materials [1]. In this framework, a formulation in two-dimensions is considered, with virtual elements having arbitrary shape. The key idea is to test the effectiveness of the VEM in dealing with highly localized strains due to material instabilities, typically exhibited by such types of materials undergoing severe loading conditions. The method is based on minimization of an incremental energy expression, [2],[3], with a novel construction of the stabilization energy for isotropic elasto-damage. Two regularization techniques, either based on a local or non-local approach, are adopted to overcome the well-known spurious mesh sensitivity problems, occurring in numerical computations when, in the presence of softening behaviour, the governing differential equations may loose ellipticity and resort to ill-posed boundary value problems. A set of numerical examples, ranging from a non-uniform tensile test, a three-point bending test on a concrete beam, up to a splitting test of a granite cylindrical specimen ,illustrate the efficiency of the proposed method.

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Vulnerability curves of masonry buildings by analyses of collapse mechanisms through PR method

Daniela De Gregorio[1], Antonino Iannuzzo[1], Antonio Gesualdo[2], Giulio Zuccaro[1,2], Maurizio Angelillo[3]

[1] *LUPT-PLINIVS Study Centre, University of Naples Federico II, Napoli, Italy*
E-mail: daniela.degregorio@unina.it, antonino.iannuzzo@unina.it

[2] *Department of Structures for Engineering and Architecture, University of Naples Federico II, Napoli, Italy*
E-mail: gesualdo@unina.it, zuccaro@unina.it

[3] *Department of Civil Engineering, University of Salerno, Salerno, Italy*
E-mail: mangelillo@unisa.it

Keywords: Masonry · Risk assessment · In-plane mechanism

Abstract. This work deals with the possibility to explore the probability of occurrence of a global in-plane mechanism by adopting Heyman's hypothesis [1] and by restricting the analysis to a given vulnerability class of ordinary masonry buildings.
Since in a recent paper [2], the identification of the correlations among structural-typologies which characterize Italian masonry buildings, possible collapse local mechanisms and ground accelerations have been investigated, the aim of the present paper is to extend this analysis by exploring the gap between first local and final global mechanism.
A numerical code, called PR Method (Piecewise-Rigid displacements Method) is used [3]. Based on an extension of the simplified model of Heyman to continuum masonry structures, the PR method reaches the solution of a kinematical problem by minimizing the potential energy of the loads in a suitable functional set, namely the set of Piecewise-Rigid displacements. On considering small displacements, the minimum problem takes the form of a Linear Programming Problem and can be solved, for large problems, in few seconds through the interior point method. With respect to a fixed vulnerability class, the probability of occurrence of a global in-plane mechanism is valued by conducting analyses by varying both structural geometry and external loads.
The objective of the paper is to investigate different damage levels for ordinary masonry buildings with the aim to assess vulnerability functions with reference to peak ground acceleration useful in the framework of risk assessment at national and regional scale, to understand the behaviour of masonry structures and to plane mitigation strategies and rehabilitation interventions.

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A discrete-to-continuum approach to the equilibrium of masonry vaults

Mariella De Piano, Valentino Paolo Berardi, Fernando Fraternali, Luciano Feo
Department of Civil Engineering, University of Salerno, 84084 Fisciano (SA), Italy
E-mail:mdepiano@unisa.it,berardi@unisa.it,f.fraternali@unisa,l.feo@unisa.it,

Keywords: Masonry, No-tension model, Adaptive FEM

A discrete element approach method to the equilibrium problem of masonry vaults is presented. The approach is developed within the Heyman's safe theorem and is based on the definition of thrust surface of a masonry curved structure, which represents an extension to 3d geometry of the funicular curve of an arch under fixed load conditions.

In particular, the static problem of the vault is set by associating the membrane continuous of a vault with a spatial truss through a non-conforming variational approximation of the thrust surface and membrane stress potential [1-3]. Polyhedral approximations of such functions are defined over triangulations of a simply-connected domain, which lies in the horizontal plane, and are assumed in the finite element formulation of the equilibrium problem.

The numerical procedure uses a predictor-corrector iterative procedure through manipulating an initial geometrical and topological solution with the stress function, under the geometry constraints of the form (the thrust surface has to be comprised between the extrados and the intrados of vault) and the concavity constraint of the stress function. The latter limitation allows to satisfy the no-tension model for the masonry [4]. The final purpose of the strategy defines a suitable topology optimization of the adopted thrust surface.

The proposed approach offers a prediction of regions exposed to crack pattern and allows estimating of a lower bound of the collapse load of existing vaults [5-7].

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Phase-field model for polarization evolution in ferroelectric materials via isogeometric collocation method

Patrick Fedeli[1], Alessandro Reali[2], Attilio Frangi[1], Ferdinando Auricchio[2]
[1] *Department of Civil and Environmental Engineering, Politecnico di Milano, Milano, Italy*
E-mail:patrick.fedeli@polimi.it, attilio.frangi@polimi.it

[2] *Department of Civil Engineering and Architecture, University of Pavia, Pavia, Italy*
E-mail:alessandro.reali@unipv.it, ferdinando.auricchio@unipv.it

Keywords: collocation, phase-field, Ginzburg-Landau equation

Ferroelectric materials are used in an increasing number of industrial applications due to their piezoelectric properties. These materials present a spontaneous electrical polarization below the Curie temperature, which can be however reoriented by applying an electro-mechanical loading. Phase-field models [1, 2] are often utilized to analyse the evolution of the polarization, which is governed by the generalized form of the Ginzburg-Landau equation [3]:

$$\left(\frac{\partial \psi}{\partial P_{i,j}}\right)_{,j} - \frac{\partial \psi}{\partial P_i} = \beta_{ij}\frac{\partial P_j}{\partial t} \tag{1}$$

being ψ the Helmholtz free energy, P_i the polarization vector and β_{ij} the inverse mobility tensor. In the Landau-Devonshire theory the free energy consists of five terms accounting for dielectric, elastic and piezoelectric properties as well as spontaneous polarization and domain wall information:

$$\psi(P_i, P_{i,j}, \varepsilon_{ij}, D_i) = \psi_{\text{Landau}}(P_i) + \psi_{\text{grad}}(P_{i,j}) + \psi_{\text{coup}}(P_i, \varepsilon_{ij}) + \psi_{\text{elas}}(\varepsilon_{ij}) + \psi_{\text{elec}}(P_i, D_i) \tag{2}$$

in which ε_{ij} is the strain tensor and D_i the electric displacement vector [1].

Typically, the space discretization in a phase field model is implemented resorting to finite difference schemes or to the finite element method. On the contrary, we propose here a collocation approach [4], based on isogeometric analysis [5]. Several benchmarks with examples taken from the literature [6] are discussed, showing the capabilities of the proposed method.

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Limit analysis and ductility check of reinforced concrete 3D frames

Giovanni Garcea[1], Domenico Magisano[1], Francesco Liguori[1], Leonardo Leonetti[1], Salvatore Sessa[2], Francesco Marmo[2], Luciano Rosati[2]
[1] *DIMES, Università della Calabria,* [2]*DIST, Università degli Studi di Napoli Federico II*
E-mail:giovanni.garcea@unical.it, salvatore.sessa2@unina.it

Keywords: yield surface approximation, limit analysis, 3D reinforced concrete frames

The safety check of reinforced concrete structures, especially when subjected to seismic loads is, still nowadays, a challenging problem. Reliable methods of analysis should take account of the complex structural behavior due to geometry, loads and nonlinear material behavior. However, many aspects of the actual structural behavior are not well known nor easy to be accounted for. For instance, external loads, including seismic ones, are statistically estimated but their variability in time is often unknown or too complex and, for this reason, often neglected. Furthermore, reinforced concrete is a complex material and its nonlinear behavior is strongly influenced by several aspects: initial deformations and cracks, history of the load-deformation process, confinement effects, initial self-equilibrated stress states, etc. Hence, a sophisticated modeling of the structural response contrasts with the poor information usually available to designers so that improving accuracy, with respect to more synthetic and essential strategies of analysis, is illusory.

In practical contexts, designers need methods of analysis able to evaluate the safety factor in a reliable way and whose results are not significantly influenced by the missing information [1]. The most synthetic methods are based on the limit and shakedown theorems [2], which require the knowledge of the geometry and the materials strength as structural data, that is quantities which are easily evaluable. However, the reliability of such direct methods is assured only when the Drucker condition on the materials is not violated, i.e. when an adequate ductility is available so as to allow the stress redistribution and the limit collapse mechanism to take place. In [3], a first step has been taken in this direction, proving that the capacity surfaces corresponding to infinite ductility coincide with the ones evaluated by accounting for the strain limits when the confinement effect on the reinforced concrete sections is that of new structures in seismic zone.

In this work, we investigate on the amount of ductility required to make the limit load a realistic safety factor for the structures. To this aim, a strain-driven analysis, which uses the cross-section yield surfaces in terms of axial force and bending moments, is implemented. The exact integration of the constitutive law along extremal paths gives a mechanical meaning to the kinematic solution and permits the ductility request control. Different structural configurations are considered to assess the effects of the structural irregularities.

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Minkowski plasticity in 3D frames: geometric evaluation of the cross-section yield surface and stress update strategy

Giovanni Garcea[1], Domenico Magisano[1], Francesco Liguori[1], Leonardo Leonetti[1]
[1] *DIMES, Università della Calabria*
E-mail:giovanni.garcea@unical.it

Keywords: yield surface approximation, return mapping, Minkowski sum

In many structural problems, the yield condition is defined by a multi-surface representation of the elastic domain boundary. This is the case, for instance, in the analysis of framed structures, where the yield surface of the cross-sections is usually evaluated as point cloud obtained by numerical experiments, which is then often approximated by means of a significant number of hyperplanes. In this context, the standard return mapping algorithm consists in an optimisation problem with multiple constraints, whose solution gets less and less efficient when the number of surfaces increases. The Minkowski sum of ellipsoids represents a valid alternative for the approximation of convex shapes such as the yield surface as shown in some recent works [1, 2, 3]. A low number of ellipsoids, whose parameters are evaluated by the least square method, can be used to describe the yield surface of homogeneous and composite cross-sections with a great accuracy. However, the use of the Minkowski sum is still not a standard tool in structural analysis because of two different difficulties: i) the least square problem can be time consuming and not robust; ii) a predictor-return mapping strategy is usually not exploited because the yield condition is not explicitly available. In this work, we show the great advantages of expressing the yield surface through a Minkowski sum, overcoming both the limitations previously discussed. Concerning the approximation, we demonstrate that the least square problem can be avoided by giving a mechanical meaning to each term of the sum, that is the contribution to the whole surface due to a portion of the cross-section. The second limitation is then overcome by parametrising the approximated yield surface in terms of the normal vector. The admissibility condition can be then easily checked by solving a nonlinear system of equations in a few unknowns, that is the components of the normal vector. Moreover, when the stress predictor is outside the domain, a return mapping algorithm can be formulated, exploiting the same parametrisation, as a small sized nonlinear optimisation problem. Both the predictor check and the return mapping problem are solved iteratively and result extremely efficient because converge in a few iterations and the number of variables involved is constant for any number of terms in the Minkowski sum. The proposed predictor-return mapping strategy is then fully exploited for an efficient nonlinear analysis of 3D frames by means of an ad hoc beam finite element.

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Orthotropic macromechanical damage model for the response of masonry structures

Cristina Gatta[1], Daniela Addessi[1]
[1] *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy*
E-mail:cristina.gatta@uniroma1.it, daniela.addessi@uniroma1.it

Keywords: Masonry, orthotropic response, damage

Development of accurate and efficient numerical procedures for the analysis of the mechanical response of masonry structures is still a challenging task. The main difficulties in modeling are related to heterogeneous nature of the material composed of units, blocks or bricks, connected through mortar joints. These latter act as planes of weakness and affect the overall collapse mechanism, which can involve the joints alone or a combined failure of mortar and masonry units. Sophisticated models based on micro and multi-scale approaches, provide reliable predictions but their applicability is limited to the analysis of small elements due to the high computation effort required. Macro-modeling technique appears as a fair comprimise between efficiency and accuracy, though the introduction of the material directional mechanical properties in the non-linear range is a hard issue. Therefore, only few models have been proposed [1].

In this work a new damage model for 2D analysis of masonry structures is presented and included into a finite element procedure. This is a modified and enriched version of that presented in [2] where the assumption of isotropic behavior held. Here, masonry is modeled as a homogeneous orthotropic material. A suitable definition of a damage matrix, written in terms of indipendent damage scalar variables, permits to capture the main failure mechanisms due to tensile states normal and parallel to bed joints, masonry crushing normal and parallel to bed joints and shear. Moreover, stiffness recovery, related to re-closure of tensile cracks, is taken into account by enriching the stress-strain constitutive law. Equivalent strain measures, defined on the masonry natural axes, drive evolution of the damage variables and settle the space where a three-dimensional damage limit surface is defined. Despite some difficulties arise to select a proper failure criteria [3], the limit surface is geometrically defined as the intersection of an ellipsoide and elliptical cone [1]. Only masonry strength properties and some additional properties derived from experimental data are required to construct the limit surface. Finally, mesh-dependency is efficiently overcome by adopting a nonlocal formulation.

Numerical applications are performed to explore the model capacity to describe masonry inelastic orthotropic behavior and a comparison with the results obtained with more sophisficated modeling and from experimental tests is also provided for masonry panels available in the literature.

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An efficient $\bar{B}$-formulation for the isogeometric analysis via Bézier projection: the case of the membrane locking

Leopoldo Greco[1], Massimo Cuomo[1] Loredana Contrafatto [1], Salvatore Gazzo[2]
[1] *Dept. of Civil Engineering and Architecture, University of Catania, Catania, Italy*
E-mail:leopoldo.greco@virgilio.it, massimo.cuomo@dica.unict.it loredana.contrafatto@unict.it

[2]*Ecole Nationale des Travaux Publics de l'Etat, Lyon, France*
E-mail:salvatore.gazzo@entpe.fr

Keywords: Membrane locking, Bézier projection, isogeometric analysis.

Mixed formulations for isogeometric analysis of structural models like beams and shells are an active areas of research, see [2, 3]. Classic $\bar{B}$ or Assumed Natural Strain formulations are in general inefficient when using B-Spline interpolations. In fact, the Gram matrix (or the collocation matrix), that has to be inverted for obtaining the assumed strain, is defined at the patch level. Therefore, it involves all the variables of the patch. As a consequence, the stiffness matrix that is obtained after the condensation is full.

In this contribution we present an efficient $\bar{B}$-formulation for isogeometric structural analysis that starts from the evaluation of the assumed strain control variables at the local level of the element (the section), as proposed in [3]. Generally, the assumed strain field so obtained is discontinuous and the resulting algorithm is affected by locking. The present formulation use a local reconstruction of the assumed strain field on the global B-Spline basis (at the patch level), based on the algorithm introduced by Thomas et al. in [1]. In this way, the inversion of the full Gram matrix is avoided and a banded stiffness matrix is obtained, whose width is just a little larger than the one given by the displacement formulation. The method allows to completely remove the membrane locking and presents the optimal rate of convergence for the L2 displacement error and the energy norm error. In addition, it is obtained the same accuracy of the full-$\bar{B}$ method, but with a much smaller computational cost, as shown in [4] for 1D Kirchhoff rod and in [5] for the 2D case of the Kirchhoff shell models.

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A new conforming isogeometric multi patch formulation: the Kirchhoff shell case

Leopoldo Greco[1], Massimo Cuomo[1], Contrafatto Loredana[1]
[1] *Dipartimento di Ingegneria Civile ed Architettura (DICAR),*
Universitá degli studi di Catania, Catania, Italy
E-mail:leopoldo.greco@virgilio.it,mcuomo@dica.unict.it, lcontra@dica.unict.it

Keywords: Conforming element, Kirchhoff shell, isogeometric analysis.

A formulation for conforming finite element passing the patch test suitable for multipatch isogeometric analysis of non polar structural models is presented. In this contribution the formulation is particularized to the case of 2D thin structural models: plates and shells. The formulation takes the move from a generalization of the rational Gregory Patch (see [1]) to the case of B-spline interpolation.

The displacement components of the model are interpolated in a rational space, indicated as CG^1 space, in honor to S. A. Coons and J. Gregory, (see [2]). Performing a change of basis from the control parameters of the B-splines, an element having as degrees of freedom the displacements of the control points and the edge rotations is obtained. The element is thus G^1 conforming. In this way the 1D-Hermitian interpolation can be generalized to the 2D-case for un-structured meshes. However, due to the presence of the rational enhancement, the symmetry of the cross second derivatives at the corners is lost, and this prevents the fulfillment of the path test.

Therefore, after assembling the global stiffness matrix, the symmetry of the second cross derivative in enforced element wise by means of a discontinuous Lagrangian multiplier field. In this way, from the conforming rational interpolation is obtained a polynomial conforming interpolation accounting for the completeness of the interpolating polynomial space, that ensure the satisfaction of the patch test. The proposed mixed interpolation is suitable for designing a conforming multi-patch isogeometric formulation as shown in this contribution.

The numerical investigations performed show that the proposed conforming formulation presents optimal rate of convergence and high robustness to the mesh distortions.

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The retrofit of masonry buildings through seismic dampers

Babak Jafarzad eslami
Civil Engineering Department, University of Ghiaseddin Kashani, Abyek, Iran
E-mail: *babak.jafarzadeslami01@universitadipavia.it,*

Keywords: Retrofit, base isolation, masonry building.

After the recent earthquakes occurred in some cities in Iran, such as Bam and Kermanshah, the engineering community was forced to pay special attention to the seismic vulnerability of traditional structures. Unreinforced masonry walls exhibit poor seismic performance under moderate and high seismic demand, due to the rapid degradation of stiffness. The development of effective techniques for the strengthening of these walls is an urgent need. The Base isolation system (BIS) provide solutions to mitigate seismic hazard. [1]

The seismic vulnerability of heritage masonry walls is assessed in this work by conducting extensive numerical studies on both unreinforced (fixed-base) and reinforced (Base Isolation System) masonry walls. In this manner, finite element modeling and analysis (using ABAQUS) are performed as a comparative study between a fixed-base masonry wall and similar base-isolated wall retrofitted with laminated rubber bearings. [2] Nonlinear time history analysis (using the actual Bam earthquake) have been recognized as a useful tool for the description of the behavior of masonry structures. Actually, they enable one to describe the pre-peak and post-peak behavior of the masonry walls. [3]

Finally, comparison of the failure modes between unreinforced and reinforced masonry walls shows a great efficiency of using the rubber bearing isolation (passive control vibration devices) for a reduction in acceleration and an increase in the structural resistance to earthquake excitations. [2]

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The effect of micro-polar rotation in 2D Cosserat solids

Lorenzo Leonetti[1], Nicholas Fantuzzi[2], Patrizia Trovalusci[3], Francesco Tornabene[2]
[1]DINCI Department, University of Calabria, Rende, Italy
E-mail: *lorenzo.leonetti@unical.it*
[2]DICAM Department, University of Bologna, Bologna, Italy
E-mail: *nicholas.fantuzzi@unibo.it, francesco.tornabene@unibo.it*
[3]DISG Department, Sapienza University of Rome, Rome, Italy
E-mail: *patrizia.trovalusci@uniroma1.it*

Keywords: Micropolar Continua; Strong Formulation Finite Element Method; Finite Element Method.

It has widely shown that the Cosserat model is able to describe homogenized continua in which particles and heterogeneity in general are described by an inner rotation termed microrotation [1-3]. Since the beginning, this additional degree of freedom has been properly introduced in reduced-dimensional structural models [4]. Many explicit solutions for Cosserat materials have been produced over years but the case, rather frequent, of orthotropic materials calls for the need of numerical investigations [1-3]. A peculiar feature of generalized continua is the presence of a material internal length [5]. The Cosserat continuum in particular, tends to behave as a Cauchy model when the internal characteristic length is close to the structural dimensions (macro-scale) but only if the material is isotropic or at least orthotetragonal [1]. Moreover, in the orthotropic case the relative rotation, which implies non-symmetries of the angular strain components, plays an important role that cannot be represented by generalized continua of other kinds (second gradient, couple stress, etc.) [2,3]. In the present work, the mechanical behaviour of Cosserat orthotropic two-dimensional block assemblies modeled as Cosserat is investigated paying attention to the material discontinuities and the scale effects. Different numerical approaches, using strong and weak form formulations, are adopted [6]. The results provided by two numerical techniques, the so-called Strong Formulation Finite Element Method (SFEM) [7] and the Finite Element Method, are compared. Convergence, stability and reliability of both numerical techniques will be discussed and advantages and disadvantages in terms of displacement/stress fields will be shown.

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Mixed Virtual Element Methods for Elasticity Problems

Carlo Lovadina[1], Edoardo Artioli[2], Stefano de Miranda[3], Luca Patruno[3]

[1] *Dipartimento di Matematica, Università di Milano, Via Saldini 50, 20133 Milano, and IMATI del CNR, Via Ferrata 1, 27100 Pavia, Italy*
E-mail:carlo.lovadina@unimi.it

[2] *Department of Civil Engineering and Computer Science, University of Rome Tor Vergata, Via del Politecnico 1, 00133 Rome, Italy*
E-mail:artioli@ing.uniroma2.it

[3] *DICAM, University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy*
E-mail:stefano.demiranda@unibo.it,luca.patruno@unibo.it

Keywords: VEM, Elasticity Problems, Mixed Methods

The Virtual Element Method (VEM) is a new technology for the approximation of partial differential equation problems. VEM was born in 2012, see [1], and shares the same variational background of the Finite Element Method (FEM).

In the framework of 2D elasticity problems, we here present a family of Virtual Element schemes based on the Hellinger-Reissner variational principle, see [2, 3]. As it is well-known, imposing both the symmetry of the stress tensor and the continuity of the tractions at the inter-element is typically a great source of troubles in the framework of classical Galerkin schemes. For example, when Finite Element Methods are employed, one is essentially led to adopt either cumbersome elements, or to relax the stress symmetry (this latter choice means that the underlying variational principle is changed). The reason for this difficulty stands in the local polynomial approximation that can not easily accomplish for both the symmetry and continuity constraints mentioned above.

We exploit the great flexibility of VEM to present alternative methods, which provide symmetric stresses, continuous tractions and are reasonably cheap with respect to the delivered accuracy. VEMs reach this goal by abandoning the local polynomial approximation concept, a feature originally used to design conforming Galerkin schemes on general polytopal meshes, see [1]. It is also worth noticing that our methods are amenable to a rigorous stability and convergence analysis.

In this talk, we detail the ideas which led to the design of our schemes, we state the theoretical results, and we present several numerical tests to assess the actual computational performance of the our approach. Finally, we discuss some possible future extensions.

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Serendipity Nodal Virtual Elements

Donatella Marini
Dipartimento di Matematica, Università di Pavia
and IMATI-CNR, Via Ferrata 1, Pavia, Italy

E-mail:marini@imati.cnr.it

Keywords: Virtual Element Method, Serendipity, Linear Elasticity

We present a variant of Nodal Virtual Element spaces, introduced in [1], that mimics the "Serendipity Finite Element Methods" (whose most popular example is the 8-node quadrilateral) and allows to reduce (often in a significant way) the number of internal degrees of freedom. When applied to the faces of a three-dimensional decomposition, this allows a reduction in the number of *face* degrees of freedom: an improvement that cannot be achieved by a simple static condensation. On triangular and tetrahedral decompositions the new elements (contrary to the original VEMs) reduce exactly to the classical Lagrange FEM. On quadrilaterals and hexahedra the new elements are quite similar (and have the same amount of degrees of freedom) to the Serendipity Finite Elements, but are *much more robust* with respect to element distortions. On more general polytopes the Serendipity VEMs are the natural (and simple) generalization of the simplicial case. Applications to linear elasticity problems [2] will be provided.

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Isogeometric collocation for the explicit dynamics of three-dimensional beams undergoing finite motions

Enzo Marino[1], Josef Kiendl[2], Laura De Lorenzis[3]
[1] *Department of Civil and Environmental Engineering – University of Florence, Via di S. Marta 3, 50139 Firenze, Italy*
E-mail:enzo.marino@dicea.unifi.it

[2] *Department of Marine Technology – Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*
E-mail:josef.kiendl@ntnu.no

[3] *Institute of Applied Mechanics – TU Braunschweig, Pockelsstraße 3, 38106 Braunschweig, Germany*
E-mail:l.delorenzis@tu-braunschweig.de

Keywords: Isogeometric collocation; Explicit dynamics; Nonlinear dynamics.

The study of isogeometric collocation (IGA-C) methods has been recently initiated in [1, 2] motivated by the idea of taking advantage from the higher-order accuracy of NURBS basis functions and the low computational cost of collocation. IGA-C is based on the discretization of the strong form of the governing equations, no numerical quadrature is needed and only one point evaluation per degree of freedom is required for the solution. A field where these attributes have a high impact is explicit dynamics, where IGA-C keeps the computational advantages of one-point quadrature methods without the need of hourglass stabilization techniques [2]. The foundations for IGA-C explicit dynamics were laid in [2], where one and two-dimensional linear elastic cases were addressed. In the present work, following the route initiated in [3, 4], we extend the development of the IGA-C method to the explicit dynamics of three-dimensional beams undergoing finite motions. Unlike in traditional nonlinear structural dynamics, in this case the configuration space involves the rotation (Lie) group $\mathrm{SO}(3)$. Exploiting the analogy with rigid body dynamics, we extend to flexible beams the explicit Newmark time integrator method for $\mathrm{SO}(3)$ proposed in [5], where it was demonstrated that the algorithm, in spite of being very simple, can attain, or even improve, the performance of existing methods. In addition, we address two main issues: the Neumann boundary conditions and the nonlinear term involving the angular acceleration, which unavoidably arises from the central difference scheme adopted. The capabilities of the proposed formulation are assessed through a series of numerical applications.

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Virtual Element Method for anisotropic materials: a constrained variational formulation for strong anisotropies in finite elasticity

Michele Marino, Blaz Hudobivinik, Peter Wriggers
Institute of Continuum Mechanics, Leibniz Universität, Hannover, Germany
E-mail: marino@ikm.uni-hannover.de, hudobivnik@ikm.uni-hannover.de, wriggers@ikm.uni-hannover.de

Keywords: Virtual Element Method, Anisotropic Materials, Constrained Variational Formulation

A wide range of engineering applications involve anisotropic materials, such as composite laminates or crystalline structures. The computational assessment of structures made up by these class of materials is often associated with requests of flexibility with regard to mesh generation and mesh shapes. For instance, polygonal or polyhedral Voronoi tessellation might be exploited for the analysis of the microstructure of polycrystalline materials. In this framework, the virtual element method (VEM) has been recently introduced [1] and it has been attracting attention in the framework of advanced modeling techniques for anisotropic material behavior [2] and homogenization of composite materials [3]. This method permits the use of polygonal/polyhedral elements where there is no need for a restriction to convex elements, nor it is necessary to avoid degeneracies such as element sides having an interior angle close to π radians in two dimensions [1]. This flexibility is paid in terms of a rank-deficiency of the energy functional due to the projection of the primary variable on a polynomial space, and hence by the need of introducing a stabilization term [4].

This work addresses the extension of VEM applications to anisotropic mechanical properties in finite elasticity. By the use of the representation theorem for tensor functions, suitable structural tensors allow to define a minimal basis of tensor invariants for the definition of constitutive equations under the respect of given material symmetries. A constrained variational formulation is introduced in order to account for strong anisotropies associated with the stiffening related to internal oriented micro/nano-structures, such as fibers or crystals. The formulation, already well-established addressing a transversely isotropic behavior [2], is here also enriched in order to account for an orthotropic response. To reach this goal, non-standard invariants that control the area change of elements in the direction along or perpendicular to material preferred orientation are introduced. The proposed formulation is tested in benchmark tests involving regular, distorted and Voronoi meshes. The Lagrange multiplier associated with the constrained anisotropic formulation is described by means of a special ansatz that allows to obtain a locking free behavior. Therefore, the proposed VEM formulation is proved to be suitable for applications to problems involving finite deformations.

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A 3D elasto-plasto-damaged model of confined concrete composites at the mesolevel

Gianluca Mazzucco[1], Nico De Marchi[1], Beatrice Pomaro[1], Giovanna Xotta[1],
Valentina A. Salomoni[1,2], Carmelo E. Majorana[1]
[1] Department of Civil, Environmental and Architectural Engineering (DICEA), University of Padova, Via F. Marzolo 9, 35131 Padova, Italy
E-mail: gianluca.mazzucco.@dicea.unipd.it, nico.demarchi@dicea.unipd.it, beatrice.pomaro@.dicea.unipd.it, giovanna.xotta@dicea.unipd.it, carmelo.maiorana@dicea.unipd.it
[2] Department of Management and Engineering (DTG), University of Padova, Stradella S. Nicola 3, 36100 Vicenza, Italy
E-mail: *valentina.salomoni@unipd.it*

Keywords: confined concrete, damage-plasticity, solid modeling.

The realistic modelling of concrete composites encompasses the definition of the correct geometrical reconstruction of samples and the development of a sound model for the service loads of interest. This work moves from pioneer studies of concrete at the mesoscale level [1] and presents an exhaustive methodology to handle the problem with the 3D continuum technique of Finite Elements at the mesoscale and for medium-high load cases, close to failure in confined conditions due to material heterogeneities. At the scale of concrete constituents it becomes important to reproduce the real particle packing, which is related to the w/c ratio and therefore, practically, to concrete workability and final strength. Also, at this scale local confinement effects are due to aggregate inclusions, which can lead to non-homogeneous plastic behaviours within the cement paste (softening or hardening) depending on the exerted confinement level.

The former aspect is addressed with a two-step process: i) by combining the laser scanner techniques with solid modelling CAD software to extract the solid model of real aggregates; ii) by developing a robust algorithm to place polydispersed ellipsoidal particles, assumed circumscribed to real aggregates, within the surrounding cement matrix; the algorithm includes an overlapping detection tool for an optimized compaction of inclusions.

The latter issue comprises the development of a damage-plasticity model for the cement paste [2] which consists in the combination of the non-associated plasticity model by Menétrey-Willam [3], where the yield surface is described in function of the second and the third invariant of the deviatoric stress tensor, with the plastic potential defined in [4]. Damage enters in compliance with the plastic-damage combination theory based on the effective stress, as a scalar isotropic variable which is function of the plastic strain. Comparisons with uniaxially loaded concrete samples fairly good prove the soundness of the model, also in terms of damage pattern mechanisms and their evolution around inclusions.

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Wave propagation in curved beam structure

Bibinur Meirbekova[1], Michele Brun[2], Vincent Pagneux[3]
[1]*Dipartimento di Ingegneria Civile ed Architettura, Università di Cagliari, Cagliari 09123, Italy*
E-mail: meirbekova@unica.it

[2]*Dipartimento di Ingegneria Meccanica, Chimica e dei Materiali, Università di Cagliari, Cagliari 09123, Italy*
E-mail: mbrun@unica.it

[3]*Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613, 72085 Le Mans Cedex 9, France*
E-mail: vincent.pagneux@univ-lemans.fr

Keywords: wave propagation, curved beam, dispersion properties.

The problem of wave propagation in curved beams is analyzed. The analysis is performed in the time harmonic regime and longitudinal and transverse flexural waves have been considered.

The equations governing the problem are reviewed from [1,2] and discussed in term of dispersion properties. It is found that four different regimes are present, associated with the propagation of selected longitudinal, transverse and coupled waves. A particular scenario with zero frequency propagation is found.

The transmission problem is solved implementing a Transfer Matrix approach which show the usual properties of symplectic spaces.

The transmission profiles show a separation between high frequency/low curvature regimes where the incident wave is practically totally transmitted and low frequency/high curvature regimes where, in addition to reflection, there is a strong coupling between longitudinal and flexural waves.

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Simulation of additive manufacturing processes: preliminary results and perspectives

Simone Morganti[1], Alberto Cattenone[2], Ferdinando Auricchio[2]

[1]*Department of Electrical, Computer, and Biomedical Engineering, University of Pavia, Pavia, Italy*

E-mail: *simone.morganti@unipv.it*

[2] *Department of Civil Engineering and Architecture, University of Pavia, Pavia, Italy*

E-mail: alberto.cattenone01@universitadipavia.it, auricchio@unipv.it

Keywords: additive manufacturing, lattice Boltzmann, finite elements.

In the last few years, Additive Manufacturing (AM) is collecting growing interest from industry and research. The most common AM technologies are the Powder Bed Fusion (PBF) and the Fused Deposition Modeling (FDM). In this work, we introduce and investigate two different simulation approaches to investigate the PBF and FDM manufacturing processes at microscopic and macroscopic scale.
On one hand, microscopic simulations have been successfully adopted to study the melt pool thermo-fluid dynamic evolution in PBF processes [1]. The study of thermal gradients and conductive-convective effects in melt pool evolution is fundamental to evaluate residual stresses, which can affect the mechanical behavior of the component. The most efficient microscopic simulation approach relies on the Lattice Boltzmann Method (LBM) [2]. In this work, using the LBM, we present some preliminary results of thermo- and fluid dynamic examples oriented to the much more complicated problem of the complete AM process simulation.
On the other hand, macroscopic simulations are, by far, the most adopted simulation strategy for AM processes. They have been used to evaluate thermal and residual stresses distributions on entire components [3] and, furthermore, to investigate parts distortions [4]. In this study, we set up a complete framework to simulate components realized with FDM technology using commercial softwares. An uncoupled thermo-mechanical simulation approach is proposed to investigate thermal and residual stresses distributions during the printing process. We propose the simulation of simple benchmark tests. The simulation results are validated with experimental tests, through the comparison between experimental and simulated part deformations.

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An isogeometric framework for erythrocyte electro-deformation simulation

Nicola A. Nodargi[1,2], Josef Kiendl[3], Paolo Bisegna[1], Federica Caselli[1], Laura De Lorenzis[2]

[1] *Department of Civil Engineering and Computer Science, University of Rome Tor Vergata via del Politecnico 1, 00133 Rome, Italy*
E-mail: {nodargi, bisegna, caselli}@ing.uniroma2.it

[2] *Institute für Angewandte Mechanik, Technische Universität Braunschweig Pockelsstraße 3, 38106 Braunschweig, Germany*
E-mail: l.delorenzis@tu-braunschweig.de

[3] *Department of Marine Technology, Norwegian University of Science and Technology Otto Nielsen 10, 7052 Trondheim, Norway*
E-mail: josef.kiendl@ntnu.no

Keywords: Isogeometric analysis, Red blood cell, Electro-deformation

Electrically-induced cell deformation experiments are receiving increasing attention as an attractive strategy for single-cell mechanical phenotyping. Due to the importance of red blood cell (RBC) deformability in pathophysiology, the focus is here on RBC electro-deformation [1].

A RBC consists in a liquid capsule enclosed by a biological membrane. The biological membrane behaves as a very thin shell, exhibiting both membrane and bending stiffness, whereas the nearly incompressible inner liquid (cytosol) implies an enclosed-volume conservation constraint [2]. In the present work, a surface shell formulation similar to the Kirchhoff–Love shell formulation discussed in [3] and accurately accounting for such volume constraint is proposed. For the electro-mechanical coupling, the mechanical forces induced by the electric field acting upon the RBC are evaluated adopting the Maxwell stress tensor approach [4]. A staggered fixed-point iteration scheme is then applied to implement the electro-mechanical (strong) coupling for simulating *in silico* RBC electro-deformation experiments.

Numerical simulations are presented for assessing accuracy, robustness and effectiveness of the proposed isogeometric framework. In particular, the attractive ability of the isogeometric formulation to avoid typical issues of the finite element method, such as mesh distortion during simulations, is highlighted.

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A regularized eXtended Finite Element framework coupled to multi-surface plasticity for wooden beams

Nicola Orlando[1], Clemens Gebhardt[2], Michael Kaliske [2], Elena Benvenuti[1]
[1] *Department of Engineering, University of Ferrara, Ferrara, Italy*
E-mail:nicola.orlando@unife.it, elena.benvenuti@unife.it

[2] *Institute for Structural Analysis, Technische Universität Dresden, Dresden, Germany*
E-mail:clemens.gebhardt@tu-dresden.de, michael.kaliske@tu-dresden.de

Keywords: XFEM, plasticity, damage, wood mechanics

Over millennia, owing to its remarkable mechanical properties, such as flexural strength combined with extreme lightness, wood has been used as a construction material. Wood exhibits an anisotropic non-linear behavior: ductile-plastic at compression, and brittle at tensile and shearing loads. In the literature, wooden structures are mainly modeled with multi-surface plasticity models [1, 2], and cracks and detachment along interfaces are simulated by means of interfacial [3] or cohesive elements [4]. As an alternative to cohesive elements, cracks can be dealt with by means of the eXtended Finite Element Method [5], where the set of standard finite element shape functions is enriched with suitable shape functions reproducing displacement jumps and crack tip singularities. This allows to describe the cracks inside the finite element mesh without re-meshing. The present contribution presents a novel elasto-damaging-plastic constitutive model with regularized discontinuities for an effective numerical simulation of wooden structures. The model couples, for the first time, the multi-surface plasticity model developed by Schmidt and Kaliske [2] with a regularized special version of the eXtended Finite Element Methods [6, 7], that, by introducing a regularization length into the model, eliminates any softening-induced mesh dependency. Numerical results obtained by means of the proposed approach are compared to experimental and numerical reference results for validation purposes.

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H(curl) finite element analysis of distortion gradient plasticity

Andrea Panteghini[1], Lorenzo Bardella[1]
[1] *Department of Civil, Environmental, Architectural Engineering and Mathematics, University of Brescia, Brescia, Italy*
E-mail:andrea.panteghini@ing.unibs.it,lorenzo.bardella@ing.unibs.it

Keywords: strain gradient plasticity; dislocation density tensor; H(curl) finite element.

We focus on the pioneering work-conjugate gradient plasticity theory proposed by Gurtin [1], whose main feature is the choice of Nye's dislocation density tensor as higher-order primal kinematic variable contributing to the recoverable behaviour. In the absence of other higher-order kinematic variables, this framework is characterised by kinematic higher-order boundary conditions which allow for discontinuity in some components of the plastic distortion, depending on the boundary outward normal. The implementation of such a theory requires a specific H(curl) Finite Element (FE) [2].

In Gurtin's terminology [1], Nye's tensor enters the theory through an addition to the free energy density called the defect energy. The form of the defect energy characterises the (energetic) size effect that the theory aims at predicting, whose magnitude depends on a material length scale entering the defect energy for dimensional consistency.

Instead, a "standard" FE implementation can be successful if the considered theory is extended by also including a dissipative higher-order stress work-conjugate to the gradient of the plastic distortion rate, such that the kinematic higher-order boundary conditions must be enforced on each plastic distortion component individually. The underlying dissipative size effect is proportional to a dissipative length scale, entering the phenomenological constitutive laws for dimensional consistency. In this enriched theory, the "standard" continuum FE has both the displacement components and the plastic distortion components as nodal degrees of freedom [3].

The foregoing switch in the kinematic higher-order boundary conditions, dependent on the adopted higher-order variables, makes it very difficult to obtain reliable FE solutions for the first theory by particularising the "standard" FE code for the enriched theory by choosing a suitably small dissipative length scale. Beside showing this, in the present investigation, we, most of all, aim at shedding light on the distortion gradient plasticity theory based on Nye's tensor only, in which also the use of an appropriately small rate sensitivity parameter to obtain rate-independent solutions within a viscoplastic framework requires some care and reveals some interesting features.

To this purpose, we focus on plane strain benchmarks relevant in small-scale plasticity, such as the simple shear of a strip constrained between bodies impenetrable to dislocations, the microbending, and a composite problem. For the simple shear problem we also provide an analytical solution for the rate-independent case neglecting istotropic hardening.

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A thermodynamically consistent interface cohesive model for the low cycle fatigue analysis

Francesco Parrinello[1], Guido Borino[1]
[1] *DICAM, University of Palermo*
E-mail:francesco.parrinello@unipa.it,guido.borino@unipa.it

Keywords: fatigue, endurance surface, damage

The present formulation is aimed to the development of an interface cohesive model able to approach the fatigue problem in a consistent thermodynamic framework [1, 2]. In order to comply thermodynamic principles, a Helmholtz free energy density function is introduced. The interface model is based on an irreversible cohesive law with loading-unloading-reloading cyclic behaviour, introduced in order to model the complex dissipative mechanisms of fatigue degradation, involving also phenomena such as crystallographic slip, frictional interactions between asperities, micro-cracking etc.

The cyclic plastic behaviour is modelled by an endurance surface, inside which nether plastic strain nor damage/fatigue degradation may take place. The endurance activation function is defined as a plastic limit condition $\phi_p = 0$. The shape of the endurance surface is ellipsoid in the traction components space. The endurance surface can translate in its space and the movement is modelled by kinematic-like internal variables.

A further damage activation function $\phi_d(Y, \chi) \leq 0$ is introduced in order to govern the evolution the damage under monotonic loading condition. In order to model the fatigue damage degradation related to the cyclic plastic behaviour, a combined plastic-damage activation condition is defined as $\phi_f := \phi_p + a\phi_d \leq 0$, where the parameter $a \geq 0$ links the damage degradation to the cyclic plastic behaviour.

The proposed formulation allows to model the damage increasing evolution during the elasto-plastic loading and unloading paths, showing positive dissipation for any inelastic loading condition. It can predict the fatigue fracture phenomenon in terms of evolving state variables; the formulation is not based on the Paris law [3] and does not consider the number of cycles as parameter of the damage activation function [4], it however allows to closely reproduce the experimental results.

Acknowledgment
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A three-dimensional phenomenological model for NiTi alloys describing functional fatigue and plasticity

Lorenza Petrini
Department of Civil and Environmental Engineering, Politecnico di Milano, Milano, Italy
E-mail:lorenza.petrini@polimi.it

Keywords: Nitinol, permanent inelasticity, cyclic loads

Shape memory alloys, in particular Nickel-Titanium alloys (NiTi), are commonly adopted in biomedical field for producing minimally invasive devices exploiting both pseudo-elasticity (like self- expandable stents, aortic valves, stent-grafts, endodontic rotating drills) and shape memory effect (like microactuators in robotic tools, micro-pumps and surgical instrumentation). These devices are subjected to cyclic loadings (due to blood pulsatility or leg or vessel movements or repeating procedures) and sometimes to very large deformations (due to the crimping procedure or physiological tortuous path or overloads) that may modify the material response inducing functional fatigue and/or plastic strains. Accordingly, the device effectiveness is limited or even completely compromised. In this work, aiming to develop a computational tool for the design and assessment of cyclically loaded biomedical devices, the phenomenological model initially proposed by Souza et al.[1], developed by Auricchio and Petrini [2], partially improved by Auricchio et al. [3], is used as a starting point for the formulation of a new constitutive model, able to consider both plasticity and fatigue. The model depends on eighteen material parameters, that can be calibrated performing the experimental tests reported in Figure 1 and measuring the quantities indicated. The model, implemented in the MATLAB code, was used to reproduce several experimental test results, taken from the literature and also ad hoc performed, in order to validate it.

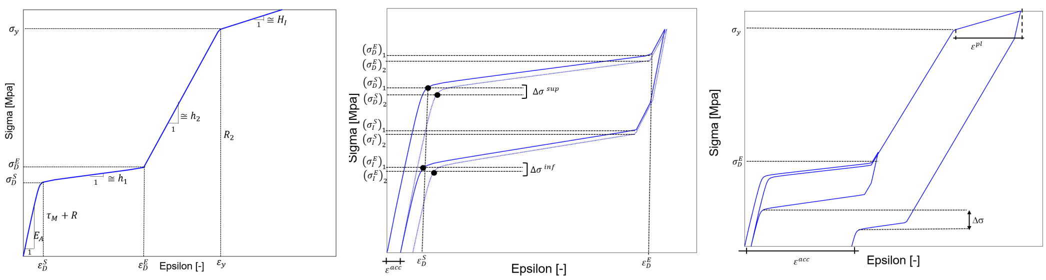

Figure 1: Experimental tests for model parameter calibration

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An Efficient Virtual Element Method (VEM) Approach for Bimaterial Systems

Marco Pingaro[1], Lorenzo Leonetti[2], Patrizia Trovalusci[1]
[1] *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Via Gramsci, 53, 00197, Rome, Italy*
E-mail:patrizia.trovalusci@uniroma1.it, marco.pingaro@uniroma1.it

[2] *Department of Civil Engineering, University of Calabria, Cosenza, Via P. Bucci, 87036, Rende, Italy*
E-mail:lorenzo.leonetti@unical.it

Keywords: Virtual Element Method, Finite Element Method, Bimaterial Systems.

Some materials used in engineering exhibit biphasics structure or they are assimilable to biphasics materials – examples of these materials are particle composite and masonry-like materials. Many studies have been devoted to the numerical implementation of computational strategy to deriving the constitutive models of such materials [5, 4]. In the present work some reference benchmarks [3], referred to an isotropic heterogeneous sample, are shown by comparing the solutions provided by the Virtual Element Method (VEM) and Finite Element Method (FEM) – given by COMSOL Multiphysics® – and the advantages of the two approaches are highlighted and discussed. The Virtual Element Method is a new promising methodology introduced by [2] and recently applied to the linear elasticity [1]. VEM is an extension of the mimetic finite difference method and it permits to use general polygons for meshes. In this work we analyse the capabilities of the VEM applied to particle composites by studying a sample that consists in a square plate with a inclusion in the centre and subjected in tension. The inclusion is modelled with a single VEM element, with low order – degree one – and high order – degree two. We perform a series of analyses with different material contrast, i.e the ratio between Youngs' moduli of inclusion and matrix.

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Advanced modeling and applications of isogeometric shells

Alessandro Reali[1,2]
[1]*Department of Civil Engineering and Architecture, University of Pavia, Pavia, Italy*
E-mail: *alessandro.reali@unipv.it*

[2] *Institute for Advanced Study, Technical University of Munich, Garching, Germany*

Keywords: Isogeometric Kirchhoff-Love shells, Composites, Fluid-Structure Interaction.

Isogeometric Analysis (IGA) is a recent simulation framework, originally proposed by Hughes et al. in 2005 [1], to bridge the gap between Computational Mechanics and Computer Aided Design (CAD). The basic IGA paradigm consists of adopting the same basis functions used for geometry representations in CAD systems - such as, e.g., Non-Uniform Rational B-Splines (NURBS) - for the approximation of field variables, in an isoparametric fashion. This leads to a cost-saving simplification of the typically expensive mesh generation and refinement processes required by standard finite element analysis. In addition, thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard finite elements in a number of applications ranging from solids and structures to fluids and fluid-structure interaction (FSI), opening also the door to geometrically flexible discretizations of higher-order partial differential equations in primal form, as well as to highly efficient (strong-form) collocation methods. In particular, this higher regularity gave "new life" to shell modeling and applications, making it possible to easily and efficiently implement (rotation-free) Kirchhoff-Love shells [2].
Within this context, this lecture focuses on some recent advances on modeling and applications of shell structures allowed by the unique IGA features. In particular, we herein focus on three interesting problems. The first one is related to an inexpensive modeling strategy for composites [3]. The proposed approach consists of an isogeometric discretization comprising a single element through the thickness and a post-processing technique able to recover an accurate out-of-plane stress state by direct integration of the equilibrium equations in strong form. We then present two novel isogeometric approaches for specific FSI problems. The first one exploits a boundary integral formulation of Stokes equations to model the surrounding flow and a nonlinear Kirchhoff-Love shell theory to model the elastic behavior of the structure [4]. The proposed method seems to be particularly attractive for the simulation of falling objects, since only the boundary representation (B-Rep) of the thin structure middle surface is needed to describe the entire studied problem. Finally, the goal of the last considered application is to show the high flexibility and potential of so-called "immersogeometric" methods to study complex problems as those typically found in Biomechanics [5]. Within this framework, FSI simulations of patient-specific aortic valve designs are successfully carried out and also compared with medical images.

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Strong and Weak Formulations for the Analysis of Arbitrarily Shaped Laminated Composite Structures

Francesco Tornabene[1], Michele Bacciocchi[1], Nicholas Fantuzzi[1]
[1]*DICAM Department, University of Bologna, Bologna, Italy*
E-mail: *francesco.tornabene@unibo.it, michele.bacciocchi@unibo.it, nicholas.fantuzzi@unibo.it*

Keywords: Differential and Integral Quadrature methods, Isogeometric analysis, Strong and Weak Formulations

A numerical approach is developed to deal with arbitrarily shaped structures. Two different methodologies are used to this aim, which are based on the Differential Quadrature and Integral Quadrature methods, respectively. These numerical methods are able to approximate both derivatives and integrals [1]. Therefore, the strong and weak formulations of the governing equations can be solved. As shown in the paper [2], these approaches are accurate, reliable and stable, when employed to obtain the mechanical response of various kinds of structures, such as plates, shells and membranes. In particular, their effectiveness is proven by means of the comparison with the analytical solutions available in the literature, both for isotropic and composite structures.

With respect to other approaches such as the Finite Element Method (FEM), the proposed methodologies are able to get the solution with few degrees of freedom. In addition, the convergence behavior is faster than the FEM.

A domain decomposition based on Isogeometric analysis is developed to analyze the mechanical behavior of arbitrarily shaped structures. The so-called blending functions are used to deal with discontinuities and distortions by means of a reduced number of elements [3, 4]. Thus, a nonlinear mapping is achieved by employing NURBS curves. According to the numerical method used in the computation, the strong and weak formulations are solved within each element. The effect of distorted meshes on the solution is investigated, as well. The numerical methods at issue are named Strong Formulation Finite Element Method (SFEM) and Weak Formulation Finite Element Method (WFEM).

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An accurate and computationally efficient phenomenological model for rate-independent hysteretic mechanical systems and materials

Nicoló Vaiana[1], Francesco Marmo[1], Salvatore Sessa[1], Luciano Rosati[1]
[1] *Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*
E-mail:nicolovaiana@outlook.it

Keywords: Phenomenological model, Rate-independent hysteresis

Hysteresis is a widespread phenomenon extensively observed in mechanical systems and materials [1, 2]. Hysteresis is usually termed rate-dependent (rate-independent) whether the rate of variation of the input does (not) influence the output [3].

In the literature, there exists a huge number of phenomenological models developed to reproduce the rate-dependent and/or rate-independent response of mechanical systems and materials. Such models are usually classified depending on the kind of equation that needs to be solved for evaluating the output variable, that is a generalized displacement or force; thus, it is possible to distinguish among algebraic, transcendental, differential, and integral models. Each phenomenological model has its own advantages and disadvantages with reference to accuracy of results, computational efficiency, computer implementation, and number of parameters.

A class of uniaxial phenomenological models has been recently formulated by Vaiana et al. [4] in order to simulate the hysteretic response of rate-independent mechanical systems and materials. The proposed formulation offers important benefits: it requires only one history variable, allows one to evaluate the generalized force of the analyzed hysteretic mechanical system or material by solving a scalar equation, and can be easily implemented in a computer program.

In this paper, a specific instance of the class, denominated Hyperbolic Model, is developed to illustrate the potentiality of the class; the presented model is able to accurately simulate the complex hysteretic response of rate-independent mechanical systems and materials, having either kinematic hardening or softening behavior, and requires a reduced number of parameters having a clear mechanical significance.

The accuracy and the computational efficiency of the developed model are assessed by carrying out some nonlinear time history analyses performed on a Single-Degree-Of-Freedom (SDOF) mechanical system having rate-independent kinematic hardening behavior, subjected either to a harmonic or to a random force. The numerical results are compared with those obtained by employing a modified version of the Bouc-Wen Model [5, 6, 7], that is one of the most used differential models in the literature.

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Phase-field modelling of the pseudo-ductile response of hybrid laminates

Roberto Alessi[1], Francesco Freddi[2]
[1] *Dipartimento di Ingegneria Strutturale e Geotecnica, SAPIENZA Università di Roma*
E-mail:roberto.alessi@uniroma1.it

[2] *Dipartimento di Ingegneria ed Architettura, Università di Parma*
E-mail:francesco.freddi@unipr.it

Keywords: hybrid composites, phase-field, variational approach

A possible strategy to provide the composite with a ductile failure response is to consider novel composite architectures where fibres of different stiffness and ultimate strain values are combined (hybridisation) [1]. In a tensile test, failure occurs first in the fibres with lowest ultimate strain inducing a stress relaxation (a decrement of stress at fixed global mean strain for the hybrid composite). The remaining high elongation fibres are proportioned to sustain the total load up to failure. By optimising the quantity and type of fibres, a ductile to failure response can be achieved.

In this work we present a 2D extension of the simplified 1D phase-field formulation [2, 4] to capture the main feature of the UD hybrid composites [3]. The brittle behaviour of the two materials is determined through the minimisation of an energy functional defined on two fields, i.e., the displacement and the damage fields, establishing an energetic competition between the elastic energy release and material damage localised in fracture surfaces. Moreover at the interface level a nonlinear cohesive interface law is assumed. Fracture mechanisms are carried out by parametric analyses of the different failure mechanisms of the laminates. The models is capable to reproduce and describe at the same time failure within the material and delamination at the interface level according to experimental observations.

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Shrinking/swelling-induced instabilities in multilayered elastic bodies

Daniele Battista[1], Paola Nardinocchi [1]
[1] *Sapienza University, Rome, Italy.*
E-mail: daniele.battista@uniroma1.it

Keywords: *instability, shrinking, multilayer*

The effect of shrinking/swelling in the stability of multilayered elastic materials has been largely investigated in the recent years as a tool to quickly morph hydrogel–based thin structures. From a stability perspective, differential shrinking/swelling have two main effects. Firstly, a change of volume modifies the geometry of the system and possibly the critical lengths involved in stability thresholds. Secondly, it may induce residual stresses in the material which can drastically affect the morphology of the thin bodies and causes mechanical instabilities. An example of this phenomenon is the swelling-induced wrinkling at the extrados of a bilayer gel beam with the softer layer on top [1], which has also been studied under transient conditions [2].
The aim of the present work is to identify the critical conditions which induce instability in structural systems consisting of softer and stiffer layers in the form of wrinkling and snapping phenomena. We borrow our methods of solution from finite elasticity via multiplicative decomposition of the deformation gradient tensor in an elastic distortional component and an elastic response component [3]. An incremental analysis is implemented to conduct a linear bifurcation analysis [4].

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A poroelastic model of bio-inspired scaffolds used for spinal fusion

Daniela P. Boso[1]

[1]*Department ICEA, University of Padova, Via Marzolo 9, Padova, Italy*
E-mail: *daniela.boso@unipd.it*

Keywords: porous media, bio-inspired scaffold, mathematical modeling.

Scientific advances in biomaterials, stem cells, growth and differentiation factors, and biomimetic environments have created unique opportunities to fabricate tissues in the laboratory from combinations of engineered extracellular matrices (scaffolds), cells, and biologically active molecules.

Within a scaffold, complex biological phenomena take place: cell seeding, cell migration and proliferation, cell differentiation, consumption of nutrients, to name a few. In this work, we combine a mathematical model [1] and laboratory experiments [2] to study the evolution of bone formation in a bio-inspired scaffold used for spinal fusion. We investigate the influence of the physiological parameters and the mechanical stimulus on the differentiation of mesenchymal stem cells into trabecular bone and, at a later stage, of trabecular bone cells into cortical bone [3].

The mathematical model is based on porous media mechanics. We consider the scaffold as composed by two main constituents: a solid phase, that is made by cells and extracellular matrix (ECM), and a liquid phase given by the interstitial fluid. The two phases include different species. In particular, in the solid phase we have the mesenchymal stem cells (MSC), the trabecular bone (TB) and the cortical bone (CB). To investigate the effects of the external loading on tissue differentiation, three different analyses are performed. In the first one, no external loads are considered, to assess the capacity of the model to match in vivo results. In the second one, a pressure is linearly increased over a time of 200 minutes, starting at 10000 minutes (about 1 week) and then is kept almost constant until the end of the simulation. In the third case the scaffold is loaded at four weeks, to study the effects of a delayed mechanical stimulus on the phenomenon.

We report the cell differentiation curves for the three cases mentioned above, supplemented with the evolution of quantities of interest, such as the effective stresses and interstitial fluid pressures. We analyze the dependence of the bone development on the mechanical environment, with a particular focus on the capability of hydrostatic effective pressure to limit the velocity of tissue differentiation during time.

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Optimal 2D auxetic microstructures with band gap

Matteo Bruggi[1], Valentina Zega[1], Alberto Corigliano[1]
[1]*Department of Civil and Environmental Engineering, Politecnico di Milano, Milano, Italy*
E-mail: *matteo.bruggi, valentina.zega, alberto.corigliano @polimi.it*

Keywords: topology optimization, homogenization, auxetic materials, band gap.

The most popular feature of auxetic structures is that they expand in the direction perpendicular to an externally exerted tension. Also, auxetic materials are of interest because of the enhanced properties related to their negative Poisson’s ratio, such as increased shear modulus, indentation resistance, fracture toughness, energy absorption [1]. The auxetic behaviour does not depend on the scale and a variety of auxetic materials and structures can be investigated, see e.g. [2].

Topology optimization has been successfully adopted to perform material design, also addressing the synthesis of extremal composites with negative Poisson’s ratio [3]. In general, optimal results consist of anisotropic microstructures where thin members are connected through weak regions ("hinges") to allow for the desired auxetic deformation. However, hinges are an issue both for manufacturing and strength.

In this contribution, an inverse homogenization approach is embedded into a volume-constrained formulation of topology optimization to design single-material 2D periodic microstructures with extreme auxetic behaviour. A suitable objective function is implemented along with a robust filtering scheme to search for isotropic microstructures that are free from any geometric singularity. A Floquet-Bloch wave approach is therefore applied to the achieved periodic cells to investigate possible band gaps characterizing the material.

Numerical simulations disclose tetra-chiral and anti-tetra-chiral auxetic lattices characterized by the arising of full/partial band gaps [4]. This feature can be tuned within the formulation by acting on the allowed volume of the microstructure or by introducing ad hoc robust constraints, as currently under investigation.

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Phase-field modelling of cracks in variably saturated porous media with application to desiccation

Tuanny Cajuhi[1], Claudio Gavagnin[2], Laura de Lorenzis[1], Lorenzo Sanavia[2]
[1]*Institute of Applied Mechanics, Technische Universität Braunschweig, Germany*
E-mail: *t.cajuhi@tu-braunschweig.de, l.delorenzis@tu-braunschweig.de*

[2]*Department of Civil Environmental and Architectural Engineering, University of Padova, Padova, Italy*
E-mail: *gavagninclaudio@gmail.com, lorenzo.sanavia@unipd.it*

Keywords: Variably saturated porous media, cracks, phase-field approach.

Porous media such as soil, rocks and concrete are of great importance in the context of civil engineering and environmental geomechanics. They consist of a solid skeleton and pores filled with fluids, e.g. air and water. Complex mechanisms of flow and transport take place within the pore network and can lead to deformation of the solid skeleton and eventually to fracture phenomena [1]. Phase-field modelling of fracture has recently emerged as an alternative to conventional approaches such as remeshing, extended finite element methods or cohesive zone modelling. The phase-field framework can be considered a special type of gradient damage modelling approach, where a diffusive approximation of the crack is taken into account and the continuous phase-field parameter is used to describe the material integrity. The essential advantages are the possibility to describe arbitrarily complicated fracture patterns without ad-hoc criteria on a fixed mesh through the solution of partial differential equations derived from variational principles [2-4]. Phase-field modelling of fracture in porous media has been addressed in some recent publications, e.g. [5], which however have only focused on the fully saturated case. Objective of this contribution is to describe fracture in partially saturated porous media [6]. The overall balance of linear momentum, the continuity equation and the phase-field evolution equation constitute a nonlinear coupled and time-dependent system of equations. We formulate the coupled non-linear system of partial differential equations governing the problem with displacements, capillary pressure and crack phase-field as unknowns. The spatial discretization is carried out with finite elements. We discuss its solution and present some relevant examples.

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Passive and active remodelling of fibre-reinforced hyperelastic solids

Jacopo Ciambella[1]
[1] *Dipartimento di Ingegneria Strutturale e Geotecnica*
SAPIENZA Universit di Roma, Via Eudossiana 18, 00184 Roma
E-mail:jacopo.ciambella@uniroma1.it

Keywords: nonlinear elasticity, transverse isotropy, active materials

Stiffness remodeling induced by a change in fiber orientation in microstructured materials, also called reorientation, is a process occurring in biological as well as active functional materials. In this contribution, we propose a model for a transversally isotropic solids in which the direction of transverse isotropy can evolve with time. This process, which we call remodelling or reorientation, is driven by an Eshelby-like torque that automatically arises from the kinematic prescriptions of the continuum and can be made dependent upon external stimuli such as growth, humidty, temperature, electrical and mangetic field. This torque causes the direction of transverse isotropy to evolve with time through a time-dependent rotation matrix $\mathbf{R}(t)$. Differently from other contributions available in the open literature [1, 2, 3], the evolution law of $\mathbf{R}(t)$ rather than postulated is derived from suitable thermodynamical restrictions imposed on the constitutive equation. The results show that, in absence of external stimuli, the stationary solutions of the remodelling equation are those rotations which yield the strain tensor coaxial with the corresponding stress tensor, which, in turn, implies that the fibres reorient towards one of the principal strain directions. This is a joint work with P. Nardinocchi.

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Un approccio numerico ad elementi finiti per la simulazione di flussi granulari tridimensionali

Massimiliano Cremonesi[1], Alessandro Franci[2]
[1] *Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano, Milano, Italia*
E-mail: massimiliano.cremonesi@polimi.it

[2] *International Center for Numerical Methods in Engineering (CIMNE), Barcelona, Spain*
E-mail: falessandro@cimne.upc.edu

Keywords: flussi granulari, Particle Finite Element Method (PFEM), $\mu(I)$ rheology

I materiali granulari densi sono presenti in molti fenomeni naturali ed in molte applicazioni industriali. Flussi di materiali granulari si trovano in ambito alimentare, farmaceutico e chimico dove i solidi, in forma di granuli, vengono miscelati, trasportati e stoccati. La capacità di prevedere il moto e la distribuzione degli sforzi in questi materiali è un prerequisito necessario per la progettazione di apparecchiature industriali. In ambito civile, frane, colate detritiche e valanghe sono ben rappresentate da materiali granulari. La loro descrizione matematica risulta tuttavia molto complessa, il che rende difficile l'utilizzo di modelli numerici per la previsione del loro comportamento.

Da un punto di vista computazionale, i materiali granulari sono stati trattati sia con approcci discreti (DEM ad esempio) sia con approcci continui (FEM e MPM). In questo lavoro si propone un approccio basato sull'ipotesi di mezzi continui in cui le equazioni tipiche della fluidodinamica vengono risolte con una reologia definita ad-hoc per il trattamento di materiali granulari. In particolare, viene utilizzata la ben nota "$\mu(I)$ rheology", una legge costitutiva che nasce da analisi sperimentali svolte sui materiali granulari [1]. Se da un lato questo modello risulta di semplice implementazione in un codice di calcolo fluidodinamico, dall'altro ne complica notevolmente la natura. Infatti, le equazioni risolventi diventano altamente non lineari e in alcuni casi anche mal poste. In questo lavoro vengono proposti degli schemi di regolarizzazione che permettono la trattazione numerica efficiente della "$\mu(I)$ rheology'.

Il Particle Finite Element Method (PFEM) [2] viene utilizzato per la soluzione numerica delle equazioni governanti. Il PFEM è un metodo Lagrangiano ad elementi finiti basato su una continua ridefinizione del dominio di calcolo che permette la trattazione automatica di superfici libere e superfici di interazione. Il metodo, validato in diversi campi di applicazione [3,4], è stato esteso per poter considerare una legge costitutiva per materiali granulari come la "$\mu(I)$ rheology'.

La formulazione proposta è stata validata con diversi esempi, sia bidimensionali che tridimensionali, in cui la soluzione numerica è stata confrontata sia con dati sperimentali che con soluzioni analitiche. L'efficacia delle regolarizzazioni proposte è stata confermata dall'accuratezza delle soluzioni ottenute.

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Simulation of VEGF receptor recruitment on ECs membrane

Valentina Damioli[1], Alberto Salvadori[2], Mattia Serpelloni[1], Gian Paolo Beretta[1], Cosetta Ravelli[3], Stefania Mitola[3]
[1] *DIMI, University of Brescia, Brescia, Italy*
E-mail:v.damioli@unibs.it, m.serpelloni002@unibs.it, gianpaolo.beretta@unibs.it

[2] *DICATAM, University of Brescia, Brescia, Italy*
E-mail:alberto.salvadori@unibs.it

[3] *DMMT, University of Brescia, Brescia, Italy*
E-mail:cosetta.ravelli@unibs.it, stefania.mitola@unibs.it

Keywords: angiogenesis, VEGFR-2, simulations

Angiogenesis, the new blood vessel formation from a pre-existing one, is an essential process of cancer growth. Many proteins play major roles in this phenomenon and we focus to the interactions among receptors, in particular Vascular Endothelial Growth Factors Receptor 2 (VEGFR-2), and their canonical and non canonical ligands, as Vascular Endothelial Growth Factors (VEGF) and Gremlin, on the endothelial cell (EC) membrane. Ligand binding to VEGFR-2 is associated with the receptor dimerization and activation, which triggers a downstream signaling pathways leading to EC proliferation. The complex VEGFR-2/ligand interacts with others transmembrane receptors, which includes $\alpha_v\beta_3$ integrins [1], responsible of cell contractility, and co-receptors, as neuropilin (NRP). Mathematical models and computer simulations, accounting for the many findings that have been provided by biologists, are able to predict conditions for angiogenesis and to identify new strategies in drug delivery.

Co-designed experiments and simulations for VEGFR-2 relocalization driven by VEGF or Gremlin has been recently proposed by the authors [2]. The model is based on continuity equations (for mass, energy and entropy), standard chemical kinetics, thermodynamics restrictions, and constitutive specifications. The governing equations in a weak form, are approximated by Finite Element and Backward Euler Methods and implemented in a in-house computer code. Numerical simulations have been validated against experimental data.

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Existence and Asymptotics for a Reaction-Diffusion-Drift Equation in the Continuum Physics of Scintillating Crystals

Fabrizio Daví
DICEA, Universitá Politecnica delle Marche, 60131 Ancona (I)
E-mail: f.davi@univpm.it

Keywords: Scintillators, Reaction-Diffusion-Drift equations, Asymptotic Convergence.

A scintillating crystal is a wavelength shifter, namely a material which converts γ- or X-ray energy into visible light. The scintillation process is a microscopic one: the incoming energy creates a population of excitation carriers (charged particles) which recombine in a dissipative and non linear manner to yield photons in the visible range [1]. We represent the excitation carrier at a mesoscopic scale with an m-dimensional vector n and we assume that the crystal can be represented as a continuum with microstructure, to arrive at a Reaction-Diffusion-Drift equation which describes the excitation carriers generation and recombination process [2]:

$$\text{div}(D(\theta)\nabla n + M(\theta)N(n)q \otimes \nabla\varphi) + R(n\,,\theta)n = \dot{n}\,; \qquad (1)$$

here θ is the absolute temperature, M and $D = k_B\theta M$ are the *Mobility* and *Diffusivity* matrices with k_B the Boltzmann constant, q is the charge number vector, $R(n\,,\theta)n$ is the non-linear *Recombination term* and $N(n) = \text{diag}(n_1, n_2, ...n_m)$. Equation (1), which is coupled with the heat equation and the Poisson equation for the *local electric field* φ, generalizes the phenomenological model of [3] and closely reminds the semiconductors equation obtained by starting from a totally different approach into [4].

In the isothermal case we obtained for (1) results about the solution global existence in time, and its asymptotic behavior by following the techniques used into [5] and [6]. We obtain, for the first time, an explicit representation of the scintillator *Decay time* and *Light yield* (which are a measure of the scintillating crystal resolution and efficiency) in terms of the equation physical parameters.

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Variational asymptotic homogenization of slender pantographic media

Francesco dell'Isola[1,2,3,4], Emilio Barchiesi[1,2,4], Luca Placidi[2,4,5]
[1]*Dipartimento di Ingegneria Strutturale e Geotecnica. Università degli Studi di Roma "La Sapienza". Roma, Italia.*
[2]*International Research Center for the Mathematics and Mechanics of Complex Systems. Università degli Studi dell'Aquila. L'Aquila, Italia.*
[3]*Dipartimento di Ingegneria Civile, Edile-Architettura e Ambientale. Università degli Studi dell'Aquila. L'Aquila, Italia.*
[4]*Research Institute for Mechanics. Lobachevsky State University of Nizhni Novgorod. Nizhni Novgorod, Russia.*
[5]*Facoltà di Ingegneria. Università Telematica Internazionale UNINETTUNO. Roma, Italia.*

E-mail: fdellisola@gmail.com (Francesco dell'Isola); barchiesiemilio@gmail.com (Emilio Barchiesi); luca.placidi@uninettunouniversity.net (Luca Placidi)

Keywords: homogenization; micro-macro identification; non-linear pantographic beam.

In the last decade, pantographic structures have proven to be an archetype in the mechanics of generalized media [1]. They are one of the simplest examples of structures whose continuum description results in a wealth of non-standard problems in the theory of higher gradient and micromorphic continua and their related mathematical challenges. Homogenization techniques are used in engineering as a mean to derive continuum models, which are in this context meant to describe synthetically at a certain scale periodic discrete structures with a characteristic length much smaller than that scale. Our starting point is a discrete-spring slender pantographic system, like that considered in [2]. By means of a variational asymptotic procedure, we derive [3] a 1-dimensional continuum model for the description of finite motions of elastic planar slender pantographic structures, whose highly exotic features are discussed. We show that, remarkably, the deformation energy density of such continuum not only depends, like the Elastica, on the Lagrangian curvature but, also, on the elongation gradient. Such 1-dimensional continuum exhibits phase transition and negative stiffness as well.

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A variational-asymptotic homogenization model for the characterization of viscoelastic materials with periodic miscrostructure.

Rosaria Del Toro[1], Andrea Bacigalupo[1], Marco Paggi[1]
[1] *IMT School for Advanced Studies Lucca, Lucca, Italy*
E-mail:rosaria.deltoro@imtlucca.it, andrea.bacigalupo@imtlucca.it, marco.paggi@imtlucca.it

Keywords: dynamic variational-asymptotic homogenization, periodic materials, viscoelasticity

A non-local dynamic homogenization technique for the analysis of a viscoelastic heterogeneous material which displays a periodic microstructure is herein proposed. The asymptotic expansion of the micro-displacement field in the transformed Laplace space allows to obtain, from the expression of the micro-scale field equation, a set of recursive differential problems defined over the periodic unit cell. Consequently, the cell problems are derived in terms of perturbation functions. Such functions depend on the geometrical and physical-mechanical properties of the material and take account into the microstructural heterogeneities. A down-scaling relation is then obtained in a consistent form, which correlates the microscopic to the macroscopic transformed displacement field and its gradients through the perturbation functions. Average field equations of infinite order are determined by substituting the down-scale relation into the micro-field equation. Based on a variational approach, the macroscopic field equation of a non-local continuum is delivered and the local and non-local overall constitutive and inertial tensors of the homogenized continuum are determined. The problem of wave propagation in case of a bi-phase layered material with orthotropic phases and axis of orthotropy parallel to the direction of layering is finally investigated. In such a case, the local and non-local overall constitutive and inertial tensors are determined analytically. Finally, in order to test the reliability of the proposed approach, the dispersion curves obtained from the non-local homogenized model are compared with the curves provided by the Floquet-Bloch theory.

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Advanced modeling of mixed-mode adhesive materials and interfaces

Rossana Dimitri[1], Giorgio Zavarise[2]
[1]*Department of Innovation Engineering, Università del Salento, Lecce, Italy*
E-mail: *rossana.dimitri@unisalento.it*

[2] *Dipartimento di Ingegneria Strutturale, Edile e Geotecnica Politecnico di Torino, Torino, Italy*
E-mail: *giorgio.zavarise@polito.it*

Keywords: Adhesive interfaces; Laminated structures; Mixed-mode debonding.

Adhesive bonded joints have attracted the attention of many industries, such as marine and aerospace, as an interesting alternative to the traditional joining methods as riveting, bolting or welding. One of the most important damage modes of adhesive joints and interfaces is related to the non-linear and irreversible debonding process, which includes the formation and propagation of interface cracks, up to the complete detachment of the adherends.

In this framework, the interfacial debonding problem is here handled through an innovative cohesive formulation, named as Enhanced Beam Theory (EBT), where the specimens are considered as an assemblage of two composite sublaminates, partly bonded together by an elastic interface. This last one is modeled with a continuous distribution of cohesive springs acting in the normal and/or tangential direction, depending on the mixed-mode condition. This generalizes the idea suggested recently in [1] for a single mode-I debonding, and extended in [2] to include mixed loading, geometry and mechanical conditions. The debonding onset and propagation is determined numerically along the weak interfaces subjected to mixed-mode conditions. The accuracy of the proposed formulation is verified against some analytical predictions and theoretical formulations available in literature [3], [4].

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Viscoelastic behaviour of polyvinyl butyral: influence of solar radiation

Mario Fagone[1], Giovanna Ranocchiai[1]
[1]*Department of Civil and Environmental Engineering, University of Florence, Italy*
E-mail: *mario.fagone@unifi.it, giovanna.ranocchiai@unifi.it*

Keywords: Polymer degradation, solar radiation, visco-elasticity.

Polymer used as interlayers of laminated glass exhibit an evident visco-elastic behaviour and are in general very sensitive to temperature. For this reasons, the problem has been faced in recent years both of identifying adequate test procedures to determine the mechanical parameters of interlayers, to determine adequate analytical and numerical procedures to well describe the mechanical response of laminated glass and to point out plain processes to quickly perform simplified analysis [1, 2, 3, 4].

Some experimental results highlight a strong influence of weathering on the properties of PVB, that is a characteristic of many polymers; this is particularly important as laminated glass is used in architecture mainly as a barrier between inside and outside and is exposed to weathering actions that can in some cases be extreme. In particular, solar radiation seems to be a cause of modification in the structure of material and consequently in the mechanical behaviour [5] and has a strong impact on clerestories, bow windows and glass façades facing south.

In order to test the influence of solar radiation over time, a non-destructive test method is necessary, that can be repeated when damage produced by solar radiation accumulates. For this reason, a procedure has been tuned in which laminated glass beams are exposed to artificial UV radiation and tested in creep at regular time intervals, to control the progression of the mechanical alteration produced.

The obtained time-displacement curves have been analysed and compared to the curves obtained by equal beams not exposed to artificial radiation. Analytical models available in the literature [4, 6] for the description of composite beams are used to reproduce the observed phenomenon; a special effort is necessary to distinguish the influence of temperature and of UV radiation, both on the elastic and on the viscous share of deformation. Numerical simulation are performed to interpret the experimental results.

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Fracture propagation induced by gas storage operations: novel crack tracking algorithms based on a visco-plastic regularization

Francesca Fantoni[1], Alberto Salvadori[1], Paul Wawrzynek[2]
[1] *DICATAM, Univeristà degli Studi di Brescia, via Branze 43, 25123, Brescia, Italy*
E-mail: francesca.fantoni@unibs.it, alberto.salvadori@unibs.it

[2] *Fracture Analysis Consultants, Inc., 121 Eastern Heights Dr., NY 14850, USA*
E-mail:washwawr@gmail.com

Keywords: Fracture mechanics, standard dissipative systems, variational formulations

Natural gas storage operations are some of the numerous industrial activities that can induce or trigger seismicity. They alter the stress state and the pressure of the pores within the Earth crust, thus creating new fractures or altering the existing frictional condition of the faults, triggering in this way new failure phenomenons. From the beginning of the twentieth century, induced seismicity has become a growing topic of interest due to its profound socio-economic implications at a national and global scale.

Nevertheless, an accurate understanding of the conditions and physical mechanisms describing the nucleation and the evolution of seismic events is still missing.

Fracture mechanics plays a crucial role in the field and accurate predictive models describing the evolution of fracture networks below ground is desirable [1].

In this regard, a novel implicit in time 3D crack tracking algorithm is shown, providing a finite elongation at each point of the crack front due to the gas pressure. It frames the problem of three dimensional quasi-static fracture propagation in brittle materials into the theory of standard dissipative processes [2].

Proposed crack tracking algorithm derives from a visco-plastic regularization of variational formulations previously established [2], moving from the usual interpretation of visco-plasticity as a regularization of the rate-independent formulation [3].

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Stress evaluation within nonlocal Finite Element Method

Paolo Fuschi[1], Aurora Angela Pisano[1],
[1]Department PAU, University of Reggio Calabria, Italy
E-mail: paolo.fuschi@unirc.it, aurora.pisano@unirc.it

Keywords: Nonlocal FEM, reduced integration, stress evaluation.

The paper deals with the stress evaluation in 2D nonlocal elastic structures discretized by displacement-based, isoparametric, 8-nodes, Serendipity, nonlocal finite elements (NL-FEs) implemented by the authors in [1]. It is pointed out how, in the numerical analysis carried on by NL-FEs models, the computed nonlocal stresses are affected by some *spurious oscillations* arising in the zones of macroscopic inhomogeneity and propagating in the surrounding areas. This highly undesirable phenomenon, which yields an unreliable solution in terms of stresses, recalls a sort of locking in FEM widely studied in the early seventies in the context of classical (local) finite element method (see e.g. [2], [3] and references therein) and, for many formal aspects, it also recalls the *stress locking* exhibited by FEs with embedded discontinuities, [4].

To overcome the above drawback, here named *nonlocal-stress-locking*, a Gauss rule with *reduced integration* is proposed in the context of nonlocal finite elements. Some general considerations and remarks on the reasons of such oscillations, strictly related to the evaluation of the nonlocal operators involved in the numerical computations, as well as on the benefit of using a reduced integration in NL-FEM will be argued in the paper. Few numerical examples and benchmark problems, for which alternative analytical solutions are available, are presented and critically discussed to highlight the remarkable improvements obtained in terms of nonlocal stress field evaluation for the considered cases. Possible future steps of the ongoing research on this theme are also drawn.

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Effective stiffness of CNT bundles under bending

Laura Galuppi[1], Gianni Royer-Carfagni[1]
[1]*Department of Engineering and Architecture, University of Parma, Italy*
E-mail: *laura.galuppi@unipr.it, gianni.royer@unipr.it*

Keywords: Carbon NanoTubes, bending response, effective stiffness.

Since their discovery, carbon nanotubes (CNTs) have been widely perceived as a very promising material, because of their unique physical and electrical properties. CNTs have also remarkable properties and qualities as structural materials, such as stiffness, toughness and strength, leading to a wealth of applications exploiting them, including advanced composites for wearable electronics and biomedical applications.

The bending response of microscopic bundles formed by close-packed arrays of CNTs has been extensively investigated by theoretical and experimental studies, which have provided evidence that the relative shearing of the constituent CNTs plays a decisive role [1]. From these results, it may be inferred that the bending stiffness of CNT bundles should fall between two limiting cases, in the language of laminated-structure mechanics usually referred to as the "layered" and the "monolithic" limit, respectively. A few simplified models have been proposed in the literature for the bending response of CNT bundles, accounting for the partial shear coupling of the constituents. However, all these approaches rely on very restrictive assumptions and present strong limitations. In particular, they consider constituent NTs of the same length of the bundle, while, in reality, these lengths are different in several orders of magnitude.

Here, the bending of a NT bundle with circular cross section and square lattice, composed by monodispersed NTs that do not run the full length of the bundle, is analytically modelled as a structured element. The model considers that the position of the constituent NTs of each row may be offset, with respect to the adjacent rows, by a given distance. Shear connection between adjacent NTs is smeared in the longitudinal direction, so that the assumed distributed bond permits a continuum analytical approach. Since the individual NTs has a very high axial stiffness, their elongation is negligible and they may be considered subjected to pure bending deformation. Furthermore, since the interaction in axial direction among NTs belonging to the same row is of secondary importance, they are free to rearrange as a result of bending deformation. The rearrangement is analytically evaluated with a variational approach.

Based on an accurate evaluation of the load state of the individual NTs composing the bundle, the internal action on the whole bundle are evaluated. Remarkably, it is demonstrated that a constant-curvature bending deformation corresponds to null axial force and constant bending moment and, consequently, that the CNT bundle behaves as a continuous homogeneous beam with constant flexural stiffness [2]. This effective stiffness turns out to be dependent on mechanical and geometrical parameters of the bundle and of the constituent NTs. The proposed model is extended to the case of circular bundles formed by a hexagonal close-packed array geometry of CNT (honeycomb lattice).

The obtained results are in good qualitative agreement with the experimental findings [1].

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A phase-field model for strain localization

Giuseppe Giambanco[1], Emma La Malfa Ribolla[1]
[1] *Department of Civil, Environmental, Aerospace, Materials Engineering, University of Palermo, Palermo, Italy*
E-mail: giuseppe.giambanco@unipa.it, emma.lamalfaribolla@unipa.it

Keywords: phase-field, strain localization.

Strain localization in quasi-brittle materials occurs when a material is subjected to a high level of mechanical solicitations and inelastic strains develop in relatively narrow zone where micro-cracks appear. The gradual evolution of the micro-cracks results in the formation of localized bands up to the development of stress-free cracks. The localized zone or plastic band is generally associated to a faster growth of strain and is characterized by inelastic phenomena such as opening and propagation of cracks, initiation and growth of voids. Conversely, outside of this zone, the material unloads elastically.
Extensive research has been carried out to address issues related to the modeling of localized deformation. A crucial point is the kinematic description of the band that has been addressed by three main models [1]. The first category of models considers the presence of a strong discontinuity in the displacement field and is typical of elastic fracture mechanics. The second approach considers the plastic band with finite thickness separated from the remaining part of the body by two weak discontinuities, as a zero-thickness interface characterized by its own tractions-displacement jumps law. Lastly, the third group considers constitutive enrichments with an internal length scale related to the width of the localization zone. Nonlocal and gradient theories that relate the constitutive behavior of a material point with those in the neighboring region, fall into this category.
Recently, phase-field models, usually adopted to describe gradual chemical changes from one phase to another, have been applied to model the transition between the fully broken and the sound material in a diffusive way. These models are characterized by the evolution of an auxiliary field (the phase field) that takes the role of an order parameter. They have been used to model brittle fracture [2] and ductile fracture [3].
The present paper presents a thermodynamically consistent formulation of the localization problem in quasi-static regime adopting the phase field approach. The introduction of the phase field variable enriches the solid kinematics, in this sense the proposed formulation can be categorized in the class of the regularized models where the plastic band is smeared on its neighboring volume with the order parameter assuming the unit value in middle surface of the band and zero value far from the same one. Several examples demonstrate the ability of the model to reproduce some important phenomenological features of brittle fracture as reported in the experimental literature.

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A variational two-phase model for micro- and macro-cracking in composites, with applications to fiber-reinforced concretes

Giovanni Lancioni[1], Roberto Alessi[2]
[1] *Dipartimento di Ingegneria Civile, Edile e Architettura, Universitá Politecnica delle Marche, Via Brecce Bianche, 60131 Ancona, Italy, E-mail: g.lancioni@univpm.it*
[2] *Dipartimento di Ingegneria Strutturale e Geotecnica, Sapienza Universitá di Roma, Via Eudossiana 18, 00184 Roma, Italy, E-mail: roberto.alessi@uniroma1.it*

Keywords: incremental energy minimization, fiber-reinforced concrete, micro-cracking

Nowadays, large efforts are devoted to the development of new structured materials and composites, which combine materials with different characteristics in order to significantly enhance the mechanical properties. In the field of structural engineering, an example is given by fiber-reinforced concretes, where fibers made of different possible materials (steel, glass, PVA, etc.) and with different shapes (smooth, hooked, twisted, etc.) are embedded in a cementitious matrix (often high-performance concretes). The resulting composite is able to sustain significant tensile loads without the need of rebars, and, in the meantime, exhibits enhanced ductility. As described in [1, 2], the effect of fibres on the tensile response is triple: *(i)* the peak stress increases up to 15 MPa; *(ii)* the composite experiences a wide stress-hardening phase associated to micro- and multi-cracking processes within the matrix; *(iii)* the final softening stage associated to the development of macro-cracks becomes longer and smoother because of the bridging effects of fibres, thus conferring improved ductility to the material.

In the present work, a variational bi-phase model is proposed, where two-phases are homogeneously distributed within the composite, which is seen as a mixture. The two phases account for a brittle material and a ductile elasto-plastic material, representative of the cementitious matrix and fibers, respectively. Damage and plastic strain fields are the internal variables accounting for possible fracture in the brittle material and inelastic strains in the ductile one. In addition, a proper interphase law is introduced as function of the relative displacements between the two phases, which accounts for a diffuse interface separating ductile and brittle material portions.

Once the system has been energetically characterised with suitable potential and dissipation functions, the evolution problem is formulated as an incremental energy minimum problem. Analytical estimates and numerical solutions are determined in the simple one-dimensional contest, which show the capability of the model in capturing the different stages of the evolution observed in experiments: the stress-hardening phase of microcracking, and the following stress-softening stage of macro-crack opening. Attention is also focused on the evolution at unloading. The effects of stiffness degradation and permanent plastic deformation are considered and compared with experimental evidences. Finally, the problem is reformulated in the two-dimensional setting, and some preliminary numerical results are proposed.

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Stress analysis around a tunnel in a gravitating poroelastic half plane

Luca Lanzoni[1], Enrico Radi[2], Andrea Nobili[1]
[1]*DIEF - Dipartimento di Ingegneria "Enzo Ferrari", University of Modena and Reggio Emilia, 41125 Modena, Italy*
E-mail: *luca.lanzoni@unimore.it, andrea.nobili@unimore.it*

[2]*DISMI - Dipartimento di Scienze e Metodi dell'Ingegneria, Università di Modena e Reggio Emilia, 42122 Reggio Emilia, Italy*
E-mail: *enrico.radi@unimore.it*

Keywords: poroelasticity; tunnel; bipolar cylindrical coordinates.

The present work deals with the mechanical behaviour of a circular tunnel embedded in a semi-infinite poroelastic half plane under gravitational body forces. Owing to the geometric layout, reference is made to bipolar cylindrical coordinates (α, β) and symmetry occurs with respect to β coordinate. The fluid flux is assumed stationary, thus making the fluid pressure p a harmonic field:

$$p(\alpha,\ \beta) = A + B\ \alpha + \sum_{n=1}^{\infty} (C_n e^{n\,\alpha} + D_n e^{-n\,\alpha})\ \cos(n\ \beta) \tag{1}$$

being C_n, D_n unkonwn arbitrary constants.

The problem is governed by the following Navier equation in terms of the displacement field $\boldsymbol{u}^s$ of the solid phase:

$$\nabla^2 \boldsymbol{u}^s + [1 + (\lambda_s - \lambda^2_{sw} / \lambda_w)/\mu)]\ \nabla\ \mathrm{div}\ \boldsymbol{u}^s + (\rho g/\ \mu)\ \mathbf{e}_1\ = n\ (1 + \lambda_{sw}/\lambda_w)\ \nabla p\ /\ \mu, \tag{2}$$

where μ denotes the elastic shear modulus of the solid phase, g is the ground acceleration, $\mathbf{e}_1$ denotes the unit vector of the vertical axis, n is the porosity and the (real and positive) parameters λ_s, λ_w and λ_{sw} characterize completely the mechanical response of the poroelastic soil according to the Bowen formalism [1].

A particular solution of eqn (2) is found in closed form by introducing a Helmholtz potential for the displacement. However such a solution does not accomplish the BCs at the ring of the tunnel and at the free surface of the half plane. Then, by following the Jeffery procedure [2], an auxiliary Airy stress function is introduced that, added to the fundamental solution, allows accomplish the BCs.

Two limit situations are considered at the ring of the tunnel. In particular, a given radial pressure acting at the ring of the tunnel (Dirichlet BCs) and a given fluid flux across the contour of the tunnel (Neumann BCs) have been considered [3]. The latter situation allows investigating the effect of a tunnel having an impermeable surface embedded in a gravitating poroelastic soil.

Results in terms of fluid pressure and stress near the rim of the hole as well as at the free surface of the half plane are analysed in detail varying both the geometric and constitutive parameters of the system.

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Asymptotic characterization of the mechanical energy transport and acoustic wave polarization in beam lattice materials

Marco Lepidi[1], Andrea Bacigalupo[2]
[1] DICCA - Università di Genova, Via Montallegro 1, 16145 Genova (Italy)
E-mail: *marco.lepidi@unige.it*

[2] IMT School for Advanced Studies Lucca, Piazza S. Francesco 19, 55100 Lucca (Italy)
E-mail: *andrea.bacigalupo@imtlucca.it*

Keywords: beam lattice, energy transport, perturbation methods.

The acoustic wave propagation through cellular periodic materials is generally accompanied by a transfer of mechanical energy. The present contribution focuses on the energy transport related to dispersive waves propagating through non-dissipative beam lattice materials with microstructure. The governing dynamic equations are reduced to the minimal space of lagrangian coordinates. The linear eigenproblem characterizing the free undamped propagation of Bloch wave is formulated. The eigensolution is completely determined to associate the real-valued dispersion functions of the propagating elastic waves with the corresponding complex-valued waveforms. First, a pair of nondimensional quantities (*polarization factors*) is presented to qualitatively distinguish and quantitatively assess the linear wave polarization, according to a proper energetic criterion. Second, a vector variable related to the periodic cell is presented to describe the directional flux of mechanical energy, in analogy to the Umov-Poynting vector related to the material point in solid mechanics. The physical-mathematical relations among the energy flux, the velocity of the energy transport and the group velocity are recognized. A multiparametric perturbation methods is employed to asymptotically approximate the wave polarization factors and the mechanical energy fluxes. Finally, all the theoretical developments are successfully applied to the prototypical beam lattice material characterized by a periodic tetrachiral microstructure.

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Influence of shear and boundary conditions on the electrochemomechanics of ionic polymer metal composites

Alessandro Leronni[1], Lorenzo Bardella[1], Luca Viviani[1]
[1] *Department of Civil, Environmental, Architectural Engineering and Mathematics, University of Brescia, Brescia, Italy*
E-mail:a.leronni@unibs.it,lorenzo.bardella@ing.unibs.it,l.viviani003@studenti.unibs.it

Keywords: ionic polymer metal composite; sensing; warping.

Ionic Polymer Metal Composites (IPMCs) are laminate small-scale structures constituted by an ionic electroactive polymeric membrane sandwiched between two metal electrodes. IPMCs are transducers applied both in sensing and actuation.

We are concerned with the modelling of IPMCs as sensors and, more specifically, energy harvesters. To this purpose, we follow the electrochemomechanical framework proposed in [1]. So far, this theory has been applied to solve boundary value problems involving simple mechanics: through-the-membrane-thickness compression [2] and flexure governed by the Euler-Bernoulli beam model [1, 3, 4].

Both the results of [5] and the IPMC sandwich structure motivate a study on the effect of shear on the IPMC electrochemomechanics. Hence, we model the IPMC by adopting the Yu-Krajcinovic structural model, characterised by a zigzag warping in which membrane and electrodes undergo independent Timoshenko-like cross-section rotations [6, 7]. The time-dependent electrochemical response is governed by a modified Poisson-Nernst-Planck system [1] nonlinearly coupled with the equilibrium equations through the membrane curvature and shear strain.

We reach our goal by focussing on relevant benchmarks, whose semi-analytical solutions, in terms of stored charge and short circuit current, are obtained by using the method of matched asymptotic expansions [8]. These solutions are validated and discussed by comparison with two-dimensional finite element analyses, with emphasis on the boundary conditions at the IPMC ends.

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On the role of phase segregation in the chemo-mechanical response of intercalating electrodes

Marco Magri[1], Alberto Salvadori[1], Buket Boz[1], Tanmay Dev[1]
[1] *DICATAM - Dipartimento di Ingegneria Civile, Architettura, Territorio, Ambiente e di Matematica, University of Brescia, via Branze 43, 25123 Brescia, Italy*
E-mail: m.magri007@unibs.it, alberto.salvadori@unibs.it, b.boz@unibs.it, t.dev@unibs.it

Keywords: Li-ion batteries, trapping-reaction, phase-segregation

Any porous electrode is a complex media made of active particles of micrometric size (as $LiCoO_2$, $LiFePO_4$, and $LiMn_2O_4$) embedded in a porous matrix formed by the mixture of carbon nanoparticles and (polymeric) binder. Modeling the response of active particles during battery operations represents a crucial aspect within a multi-scale modeling approach [1].

Insertion and extraction of Li ions in active particles involve a wide range of electro-chemo-mechanical phenomena. Among those, changes in lithium content cause a series of phase-transitions and mechanical deformations.

In this contribution the impact of lithium trapping on the response of active particles is investigated through theoretical and numerical analysis [2]. Preliminary results show that a chemical reaction of trapping may induce a non-convex profile in the free energy density, as customary for phase-segregating materials. Moreover, the transient response of a single particle upon lithiation and delithiation shows that the trapping process is localized in a narrow zone that may identify a diffuse phase-interface.

Based on these evidence, the first-order phase transition occurring in $LiCoO_2$ is simulated through Li-ions immobilization. Even though interface energies are not accounted for, the present approach allow us to simulate a chemical response in agreement with the measured phase-diagram [3]. In addition, the mechanical response of the electrode is investigated by accounting for the chemical strains associated with the insertion and extraction of Li ions.

In order to validate the present model, the chemo-mechanical response of a single spherical particle is simulated numerically through Finite Element Analysis. The influence of material parameters on the lithium profiles and on the state of stress is investigated for different boundary conditions. The response of a complex multi-particle electrode microstructure is simulated as well. The impact of phase-segregation on the electro-chemo-mechanical response of the electrode compound is evaluated for different C-rates.

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Mixed-mode interface fracture of beam-like geometries: length scales and mode mixity angle

Roberta Massabò, Alessandro Cavalletti
[1]*Department of Civil, Chemical and Environmental Engineering, University of Genova, Genova, Italy*
E-mail: *roberta.massabo@unige.it, 3892934@studenti.unige.it*

Keywords: cohesive-zone model; Linear Elastic Fracture Mechanics; modulus mismatch

Sandwich and layered composites are widely used in many industrial sectors, e.g. aerospace, naval, civil and wind engineering. Interface fracture (delamination or disbond) is one of the dominant failure mechanisms and recent failures/malfunctions in aerospace components and wind turbines can be attributed to the unstable propagation of interface cracks [1]; delamination fracture also dominates the post-failure response of laminated glass, which is extensively used in automotive/civil applications [2]. Different problems are currently being investigated both in basic research (e.g. mode mixity [3]) and in applied research (e.g. standard test methods [4]).

This work deals with fracture mode partitioning in layered beam-like geometries. A phase angle can be defined as a measure of the amount of mode II to mode I loading. The phase angle is an essential factor in the formulation of failure criteria and the characterization of the fracture properties. We aim at establishing a relation between mode partition solutions obtained through linear elastic fracture mechanics [5] and through cohesive-zone modeling [3], with focus on problems characterized by modulus mismatch (Dundurs' parameter $\beta \neq 0$ [6]); for these problems LEFM solutions depend on a reference length. We will investigate, using computational and semi-analytical methods, the role played by the characteristic cohesive length-scales of the problem. In thin bodies, the length scales are material/structures properties which scale with the thickness of the body [7,8]; this is expected to influence the response and prior conclusions [3].

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Un metodo Lagrangiano Esplicito per la simulazione di Problemi Tridimensionali di Interazione Fluido-Struttura.

Simone Meduri[1], Massimiliano Cremonesi[1], Umberto Perego[1]
[1] *Dipartimento di Ingegneria Civile e Ambientale, Politecnico di Milano, Italia*
E-mail: massimiliano.cremonesi@polimi.it

Keywords: FSI, Lagrangian PFEM-FEM, Explicit Solver

In questo contributo si presenta un approccio Lagrangiano esplicito per la simulazione di problemi tridimensionali di interazione fluido-struttura. Il dominio nella partizione fluida è modellato attraverso una versione esplicita del Particle Finite Element Method [1], basata sull'ipotesi di fluido debolmente comprimibile [2]. Per il dominio solido si impiega invece il software commerciale a Elementi Finiti Abaqus/Explicit. Tale scelta permette di includere automaticamente nel modello strutturale tutte le avanzate funzionalità di Abaqus, tra cui un'ampia libreria di Elementi Finiti e legami costitutivi. L'algoritmo di Gravouil e Combescure [3], appartenente alla famiglia dei Metodi di Decomposizione dei Domini, è adottato come schema di accoppiamento. Esso permette l'impiego di discretizzazioni spaziali e temporali non conformi nei due domini fluido e solido, per ottimizzare l'efficienza di solutori espliciti che coinvolgono materiali differenti. Il conseguente problema di interazione all'interfaccia è composto da equazioni completamente disaccoppiate nel caso di mesh conformi nei due domini e debolmente accoppiate nel caso di mesh non conformi. Ne deriva quindi un solutore globale esplicito che appare particolarmente interessante per applicazioni ingegneristiche a scala reale caratterizzate da un alto grado di nonlinearità e/o una dinamica veloce. L'adozione di una descrizione Lagrangiana è inoltre vantaggiosa per problemi con flussi fluidi a superficie libera o con grandi spostamenti del dominio solido [4].
La descrizione Lagrangiana del PFEM necessita frequenti triangolazioni ogni volta che la mesh risulti troppo distorta e per questo adotta l'efficiente algoritmo di generazione della mesh di Delaunay. Nel caso tridimensionale però, questo algoritmo può generare tetraedri di bassa qualità che causano passi temporali dell'integrazione esplicita estremamente ridotti, facendo crescere inaccettabilmente i tempi di calcolo. Per questo motivo è stato sviluppato un nuovo algoritmo di regolarizzazione della mesh mirato ad aumentare la dimensione del passo di integrazione temporale. Tale algoritmo risulta estremamente efficiente perché basato su una analogia elastica che permette di sfruttare l'architettura del solutore fluido, ottenendo un algoritmo esplicito e parallelizzabile.
Numerosi confronti di validazione con risultati analitici, numerici e sperimentali presentati in letteratura hanno dimostrato l'accuratezza e la robustezza dell'approccio adottato.

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Material parameter identification for the human cornea

Andrea Montanino[1], Maurizio Angelillo[2], Anna Pandolfi[1]
[1]*Dipartimento di Ingegneria Civile ed Ambientale, Politecnico di Milano, Italy*
E-mail: {*andrea.montanino, anna.pandolfi}@polimi.it*

[2] *Dipartimento di Ingegneria Civile, Università degli Studi di Salerno, Italy*
E-mail: *mangelillo@unisa.it*

Keywords: patient-specific, cornea biomechanics, predictive numerical analysis

In the view of supporting with numerical calculations refractive surgery interventions on the human cornea, in particular anterior surface ablation, the availability of patient-specific material models is very important, since refractive outcomes are strongly affected by individual properties.

With this study we aim at defining proper material models for the human cornea and the range of variability of the corresponding material parameters. We rely on the availability of a sufficiently large set of clinical data concerning eye treated with laser ablation procedures and use finite element simulations to identify material model and parameters [1].

Clinical data refer to the purely geometrical data (i.e., a cloud of points defining the anterior and posterior surface of the cornea) of two different configuration of the same eye: the preoperative and the postoperative configurations, together with the physiological intraocular pressures (IOP) at both configurations. Geometrical information is used to produce a finite element discretized model of pre and post-operative corneas.

The acquired geometries correspond to stressed states that represent the mechanical response of the cornea to the IOPs. A correct finite element analysis requires the identification of the unstressed configurations corresponding to zero IOPs. The availability of two distinct geometries for the same eye (therefore, once the material model has been selected, for the same material properties) allows for the definition of a numerical iterative procedure able to identify at once the unstressed geometry and the material parameters. The driving ansatz is that, in the unstressed configuration, the posterior surface of the cornea remains the same in the pre and post-operative state [2].

We consider several material models, from a linear elastic isotropic material to very sophisticated fiber reinforced models that account for stochastic distribution of the collagen fibrils observed in the cornea [3]. Results are encouraging, and reveal that a simple isotropic material is not able to provide the complex configurational and refractive changes induced by surgical reprofiling of the cornea.

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On 2D and 3D band-gap structure in cellular locally resonant materials

Marco Moscatelli[1], Raffaele Ardito[1], Larissa Driemeier[2], Claudia Comi[1]
[1]*Department of Civil and Environmental Engineering, Politecnico di Milano, Milan, Italy*
E-mail: *marco.moscatelli1@mail.polimi.it,{raffaele.ardito, claudia.comi}@polimi.it*

[2] *Department of Mechatronics, University of São Paulo, São Paulo, Brazil*
E-mail: *driemeie@usp.br*

Keywords: acoustic metamaterials, dispersion spectrum, finite element analyses.

Locally resonant acoustic materials (LRAM) are periodic media, widely studied in view of their peculiar properties with respect to wave propagation. Indeed, the dispersion spectrum of LRAM exhibits bandgaps, i.e. frequency ranges in which wave propagation is prohibited, that depend on the vibrational features of its components: therefore, it is possible to obtain bandgaps with a central frequency which is well below the frequency of the wave associated to the cell dimension of the periodic arrangement. Such a behaviour has been interpreted as a "negative mass" effect that can be fruitfully exploited to get wave filters at relatively low frequency [1].

In most cases, LRAM are represented by three-phase materials, e.g. a high density material with an elastic coating, embedded in a rigid matrix [2]. In the framework of impact absorbers for crashworthiness of lightweight vehicles, Comi and Driemeier [3] have proposed the adoption of a square lattice of aluminium cells, filled by a polymeric foam, with circular lead inclusions. In that case, the study was focussed on long extruded cells that can be effectively studied by means of plane models.

In the present communication, we consider several possible variations in order to optimize the LRAM proposed in [3]. More specifically, we examine critically the differences between plane models, with the presence of decoupled in-plane and out-of-plane modes, and full 3D models. To that purpose, we analyse various 3D models, with different out-of-plane extensions. The 3D and 2D band structures are obtained by means of the execution of numerical analyses on the basis of the finite element method. The dispersion analyses necessitate of specific provisions in order to introduce the Bloch-Floquet periodicity conditions on the unit cell. The comparative examination of the achieved results leads to interesting conclusions on the interplay between in-plane and out-of-plane stiffness for cellular materials with limited extrusion length. Such knowledge of the mechanical behaviour of 3D LRAM represents the basis for the development of new layouts, e.g. by changing the shape of the unit cell and of the inclusions.

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Transition waves in Rayleigh-type flexural systems

Michael Nieves[1,2], Marta Garau[2], Michele Brun[1], Giuseppe Fraghì[1]
[1]*Dipartimento di Ingegneria Meccanica, Chimica e dei Materiali, Università degli Studi di Cagliari, Cagliari, Italy*
E-mail: m.nieves@keele.ac.uk, m.brun@unica.it
[2] *School of Computing and Mathematics, Keele University, Keele, UK*
E-mail: m.garau@keele.ac.uk

Keywords: dynamic fracture, mass-beam systems, rotational inertia, Wiener-Hopf technique.

Phase transition processes in mass-spring systems have been extensively studied in the literature with the approach developed in [1]. Recently, several articles have appeared that consider such processes in flexural systems [2,3,4].
In comparison with mass-spring systems, flexural structures allow for the study of a greater range of loading scenarios and physical parameters which influence the response of a structure. They may also describe failure propagation in civil engineering systems.

In this talk, we consider the role of rotational inertia in the process of phase transition in a one-dimensional flexural system.
This process is assumed to occur with a uniform speed that is driven by feeding waves carrying energy produced by an applied oscillating moment and force. We show that the problem can be reduced to a Wiener-Hopf equation via the Fourier transform.
The associated solution is presented and from this we identify the dynamic behaviour of the system during the transition process.
The minimum energy required to initiate the phase transition process with a given speed is determined and it is shown there exist parameter domains defined by the force and moment amplitudes where the phase transition can occur.
The influence of the rotational inertia of the system on the wave radiation phenomenon connected with the phase transition is also discussed.
All results are supplied with numerical illustrations confirming the analytical predictions.

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A unified variational approach to fracture in heterogeneous materials and composites

Marco Paggi[1], Valerio Carollo[2], Mauro Corrado[3], José Reinoso[4]
[1] IMT School for Advanced Studies Lucca, Lucca, Italy
E-mail: *marco.paggi@imtlucca.it*

[2] Dipartimento di Ingegneria Civile, Ambientale e Meccanica, University of Trento, Trento, Italy
E-mail: *valerio.carollo@unitn.it*

[3] Dipartimento di Ingegneria Strutturale, Edile e Geotecnica, Politecnico di Torino, Torino, Italy
E-mail: *mauro.corrado@polito.it*

[4] School of Engineering, Universidad de Sevilla, Seville, Spain
E-mail: *jreinoso@us.es*

Keywords: phase field model of fracture, cohesive zone model of fracture, finite element method.

A unified variational approach to brittle fracture in the bulk and quasi-brittle delamination along pre-existing interfaces is herein proposed, by coupling the phase field approach to brittle fracture and the cohesive zone model. The resulting finite element simulation framework can be used to predict complex crack patterns in laminates involving both interlaminar and translaminar cracks (Fig. 1). Moreover, the extension of the phase field approach to anisotropic materials with orientation-dependent elastic properties and fracture toughness enables the study of the competition between intergranular and transgranular crack growths (Fig. 1).

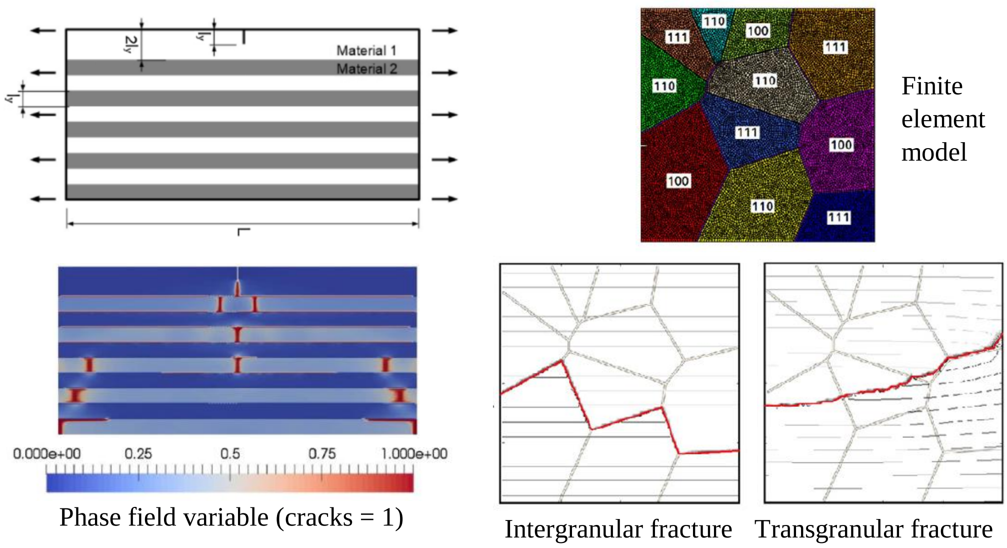

Fig. 1: simulation of complex crack patterns in laminates and polycrystalline materials.

A novel computational model to study complex crack paths in dynamic fracture of heterogeneous materials

Mauro Corrado[1], Valerio Carollo[2], José Reinoso[3], Marco Paggi[4]
[1] Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Torino, Italy
E-mail: mauro.corrado@polito.it

[2] Department of Civil, Environmental and Mechanical Engineering, University of Trento, Trento, Italy
E-mail: valerio.carollo@unitn.it

3 Elasticity and Strength of Materials Group, School of Engineering, Universidad de Sevilla, Seville, Spain
E-mail: jreinoso@us.es

4 IMT School for Advanced Studies Lucca, Lucca, Italy
E-mail: marco.paggi@imtlucca.it

Keywords: dynamic fracture, phase-field, cohesive model.

Crack branching is triggered when, during crack propagation, the energy release rate results much higher than the crack resistance. This is typical during the stage of unstable crack propagation. Moreover, in a dynamic load framework, it can be also triggered by the presence of an interface. The experiments performed by Parab and Chen [1] show that when a crack impinges on an interface the interaction between crack propagation and interface delamination can trigger the crack branching. In [1], this problem has been experimentally analyzed with different interface thicknesses.

In the present work, the occurrence of this phenomenon is predicted extending the novel computational model developed by Paggi and Reinoso [2] to the dynamic regime. This computational model roots its basis on the phase field model for brittle fracture. The key novelty concerns the formulation of an interface element, equipped with a tension cut-off cohesive zone model, coupled with the phase field model. The coupling is based on the hypothesis that the stiffness of the interface is degraded by the damage of the surrounding bulk material. The enhancement of this model to the dynamic framework is validated by benchmark numerical examples. Moreover, a comparison with the experimental results obtained in [1] is shown.

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Numerical modelling of porous media: from freezing process kinetics to mechanical effects

Francesco Pesavento[1], Dariusz Gawin[2]
[1]*Dip. di Ingegneria Civile, Edile e Ambientale, Università di Padova, Padova, Italy*
E-mail: *francesco.pesavento@dicea.unipd.it*

[2] *Dept. of Build. Phys. and Build. Mat., Łódź University of Technology, Łódź, Poland*
E-mail: *dariusz.gawin@p.lodz.pl*

Keywords: freezing process, super-cooling, multiphase porous material.

Freezing-thawing cycles and the related mechanical effects, are one of the most significant causes of durability problems for concrete structures.

These physicochemical processes are very complex due to the interactions between different phases and/or components, the inner microstructure of the material (particularly fine and well developed in the case of cementitious materials) and the fact that phase changes take place in a confined environment. This results in an alteration of the standard freezing and fusion points of water, i.e. the limits we can observe for bulk water. For instance, freezing of water in a pore network, develops in a range of temperatures below the triple point. This freezing point depression is known as undercooling process, which leads also to a sort of hysteresis between the freezing and the melting stages (because the curvature of the ice-water interface is different during two stages [1,2]). It is possible to observe also the so-called supercooling phenomenon, i.e. the first freezing of water at temperature below zero, due to the uncertainties of nucleation.

Considering what described above, water freezing and ice melting in partially saturated media are processes very difficult to model numerically. In this work we present a coupled model for the numerical modelling of heat and mass transport in partially water saturated porous media, exposed to freezing – thawing processes and the related mechanical effects. Water phase change is modelled by means of a kinetic, non-equilibrium approach with capillary pressure as the state variable. This approach is derived from the Thermodynamics of porous media [2] and is able to reproduce the cryo-suction [2], supercooling and undercooling phenomena considering in such a way the hysteresis of ice content vs. temperature during the phase change, what results also in hysteresis of material strains. The latter ones are calculated here by means of appropriately formulated effective stress principle considering crystallisation pressure in partially saturated materials.

Some relevant numerical cases will be presented to show the effectiveness of the formulated model together with a parametric study aimed at showing the importance and the role of different variables involved in the freezing-thawing process.

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An application of variational methods in strain gradient damage and fracture mechanics

Luca Placidi[1], Emilio Barchiesi[2], Anil Misra[3]
[1] *Università Telematica Internazionale Uninettuno, Rome, Italy E-mail:luca.palcidi@uninettunouniversity.net*

[2] *Università degli Studi di Roma "La Sapienza", Rome, Italy E-mail:barchiesiemilio@gmail.com*

[3] *The University of Kansas, Lawrence, KS, USA E-mail:amisra@ku.edu*

Keywords: Damge, fracture, strain gradient

Localized deformations are often encountered in engineering applications and lead to stress concentrations [2], damage and fracture mechanisms. Thus, a scalar damage field, ranging from zero to one, is generally introduced to describe the internal state of structural degradation of the material. Besides, in order to control the size of localization regions, some characteristic lengths, in the constitutive model, must be introduced and, to penalize deformations that are too localized, non-local terms are used in the internal deformation energy [3]. Damage and fracture phenomena that are here intended to be modeled are clearly of irreversible nature. In order to deal with dissipation of energy, as damage increases, a dissipation term is considered in the deformation energy. Therefore, in order to use the principle of least action, we show a variational inequality formulation, that leads to Partial Differential Equations (PDEs), Boundary Conditions (BCs) and Karush-Kuhn-Tucker (KKT) conditions. We remark that KKT conditions are associated to an explicit damage field evolution. The framework is that of 2D strain gradient damage mechanics, where the elastic strain energy density of the body has been assumed to be geometrically non-linear and to depend upon the strain gradient.

The dependence of Lam and second gradient elastic coefficients with respect to damage is not prescripted by the variational principle and need to be experimentally identified. A novel discussion on this point will be done [3]. Generally, these constitutive coefficients are all assumed to decrease as damage increases and to be locally zero if the value attained by damage is one. This last situation is associated with crack formation and/or propagation [1].

A numerical technique based on commercial softwares will be presented and discussed for a couple of exemplary problems, where a discussion will be performed as some constitutive parameters are varying, with the inclusion of mesh-independence evidence. Finally, the case of an obliquelynotched rectangular specimen subject to monotonous tensile and shear loading tests and brittle fracture propagation is discussed.

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H–M coupling for localized erosion phenomena in porous media

Andrea Francesco Rotunno[1,3], Carlo Callari[2], Francesco Froiio[3]
[1] *DICII, University of Rome "Tor Vergata", Rome, Italy*
E-mail:a.rotunno@ing.uniroma2.it

[2] *DiBT, University of Molise, Campobasso, Italy*
E-mail:carlo.callari@unimol.it

[3] *LTDS, Univ Lyon, École Centrale de Lyon, Lyon, France*
E-mail:francesco.froiio@ec-lyon.fr

Keywords: localized erosion; finite elements, discrete elements; lattice Boltzmann method

A number of hydro-mechanical erosion processes of particular interest in geomechanics refer to situations in which a porous domain is eroded in the vicinity of a free-flow region under the action of the outward seepage flow across the relevant interface. Such processes, which we refer to as normal hydro-erosion processes, are being studied for example in relation to the backward erosion mechanism responsible for the propagation of piping erosion in earth dams and levees [1]. Firstly, we present a finite element formulation able to model both the processes observed in backward piping, i.e. the upstream oriented propagation of erosion pipes and their simultaneous expansion within the hydraulic work. The numerical implementation is based on a new analytical model for the localized erosion along a line propagating in a poro-elastoplastic solid [2]. In this line, a conduit with evolving transverse size is embedded, which conveys a multi-phase flow fed by the solid and fluid masses exchanged by the erosion line with the surrounding porous medium. The two systems (i.e. porous medium and the pipe) share the same displacement- and pressure field. On the contrary, different fields are considered to describe the relevant flows, which are assumed as laminar in the porous medium and turbulent in the pipe. These flows drive pipe propagation and enlargement, respectively, as modeled by means of proper erosion kinetic laws. We propose the application of the numerical method in the analysis of the stability of excavations supported by diaphragm walls. Finally, we present a 2-D discrete numerical model of the aforementioned backward erosion mechanism [4], which is based on the coupling of DEM with the Lattice Boltzmann Method (LBM), for the description of the solid- and the fluid phase [3], respectively. A micromechanical analysis of the results provided by these models is presented: we focus on the validation of previously conjectured degradation mechanisms (e.g. localized damage and corresponding increase of porosity) and seek for the main parameters controlling the erosion kinetics.

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A novel visco-plastic model for granular materials at finite strains

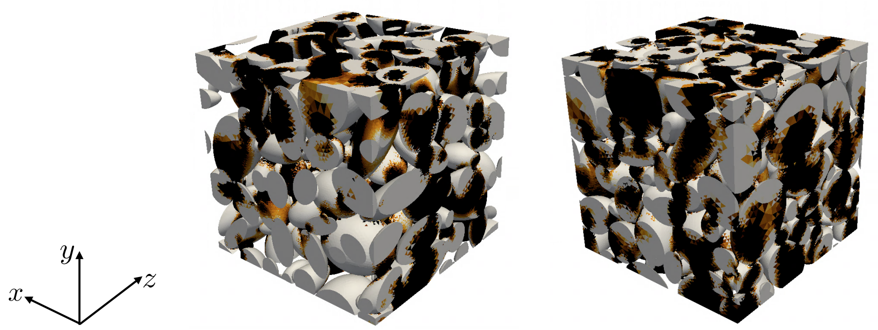
Alberto Salvadori[1]
[1] *DICATAM, Università di Brescia, Brescia, Italy*
E-mail: alberto.salvadori@unibs.it

Keywords: granular materials, poro-viscoplasticity, finite strains

A novel finite strain poro-viscoplastic model for granular materials is proposed, following [1]. This model accounts for rate-dependence, elasto-plastic coupling, pressure sensitivity, and transition to full solid state.

As recently proved in [2], the effective mechanical properties of cold compacted powders are strongly influenced by the morphology of the arrangement, in a non trivial way that depends significantly on the micro-structural interactions between the particles and on their ability to distribute the stress field through the volume. These features and ways to incorporate them in the phenomenological poro-viscoplastic model will be discussed here, too.

The has been implemented, verified, and validated against experimental analyses available in the literature for copper powder compounds. Applications in pharmaceutics will be investigated as well.



(a) (b)

Figure 1: *Stress percolation paths in the RVEs at 65% volume fraction (a) and 75% (b) in uniaxial compression tests. In the elements painted in black the magnitude of the Cauchy stress, σ_{yy}, in the loading direction is larger than the average stress in the same direction. The gray colored elements have a level of Cauchy stress in the direction of the load less the 50% of the average Cauchy stress in the same direction.*

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A degrading Bouc-Wen model with damage

Marialuigia Sangirardi[1], Daniela Addessi[1], Domenico Liberatore[1]
[1] *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Rome, Italy*
E-mail:marialuigia.sangirardi@uniroma1.it, daniela.addessi@uniroma1.it, domenico.liberatore@uniroma1.it

Keywords: Bouc-Wen-type models, damage, degradation

Hysteresis is a well known phenomenon, affecting behaviour of many different materials. Response of structural systems subjected to seismic loads, for example, is characterized by frequent excursions in the inelastic range. This is accompanied by energy dissipation due to hysteresis cycles in which strength, stiffness or both deteriorate progressively and, in some cases, arise pinching effects.

Bouc-Wen [1] model is a multivariate smooth hysteretic model and, in the context of endochronic theory, first proposed by Valanis, is undoubtedly one of the most popular in structural dynamic. It labels an entire class of hysteretic models and owes its large diffusion to its mathematical tractability, which has made it one of the most suitable tool for random vibration analyses. Moreover, Bouc-Wen model is also capable of reproducing a wide variety of hysteresis shapes, governed by a relatively small set of parameters. Baber and Wen extended the model to include stiffness and/or strength degradation introducing its dependency on the hysteretic dissipated energy. Baber and Noori [2] further extended it by incorporating pinching. Literature of past recent years has, however, highlighted some limits of the formulation. Its thermodynamic admissibility is a well-recognized problem, together with the local violation of Drucker's or Illiushin's postulates of plasticity in case of short unloading-reloading paths. Simple numerical applications have also shown that the dissipated hysteretic energy function is non monotonic. All these issues have prompted a lively debate on the topic and many authors have provided modifications to overcome above mentioned drawbacks.

In the light of the same motivations, in this work a revised Bouc-Wen model with damage is presented. A single scalar variable D has been introduced with the aim of reproducing the effects of both strength and stiffness deterioration, maintaining a rather simple mathematical framework. It depends, in the spirit of the familiar previous formulations, on the dissipated energy, whose relation with the other variables involved in the problem will be discussed.

The model proves to be able to reproduce hysteretic response of degrading systems in case of constant and varying amplitude of the displacement input and, under the hypothesis of $\beta \geq \gamma$, is thermodynamically admissible. Moreover, dissipated energy and damage are bounded, monotonic functions. In this context, a proper arrangement in series and in parallel of a negative stiffness and a non-dissipative device respectively, leads the way towards a broad range of applications, i.e. when pinching effects become non negligible, for materials which exhibit high initial stiffness, or when hysteretic cycles become more *s-shaped*. This revised Bouc-Wen model has been included in the formulation of a force-based beam macroelement for the analysis of masonry structures [3]. Finally, the results of some relevant applications are shown and compared with experimental outcomes.

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Phononic materials for the passive control of Bloch waves

Francesca Vadalà[1], Andrea Bacigalupo[2], Marco Lepidi[1], Luigi Gambarotta[1]
[1]*DICCA, University of Genoa, Italy*
E-mail: francesca.vadala@edu.unige.it, marco.lepidi@unige.it, luigi.gambarotta@unige.it

[2]*IMT School for Advanced Studies, Lucca, Italy*
E-mail: andrea.bacigalupo@imtlucca.it

Keywords: Phononic material, Bloch wave propagation, passive control.

Periodic materials can be characterized by a beam lattice microstructure [1, 2, 3]. The propagation of elastic waves through these materials can be studied by analytical formulations, in combination with the Floquet-Bloch theory. The present work deals with different analytical formulations developed either by means of high-fidelity descriptions of the heterogeneous microstructure or by virtue of continualization techniques in equivalent micropolar homogeneous continua [4, 5]. The band structure of periodic materials can alternatively be approximated through high frequency multiparametric perturbation techniques [6]. The dispersion spectra corresponding to different formulations are compared and discussed. Attention is focused on band gaps possibly occurring in the low frequency range, suited to let periodic materials serve as phononic filters for Bloch waves. The parametric conditions for the existence of band gaps are discussed for a particular case study. Alternatively, the possibility to open a band gap centered at a certain target frequency is sought for by enlarging the minimal space of constitutive parameters [7]. For a suited parameter combination, the potential of phononic filters in passively controlling the forced wave propagation is verified.

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Spectral decomposition for the constitutive modeling of distributed fiber-reinforced tissues

Marcello Vasta[1], Alessio Gizzi[2], Anna Pandolfi[3]
[1] *Universitá di Chieti-Pescara, Viale Pindaro 42, Pescara, Italy.*
E-mail: mvasta@unich.it
[2] *University Campus Bio-Medico of Rome, Via A. del Portillo 21, 00128 - Rome, Italy.*
E-mail: a.gizzi@unicampus.it
[3] *Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 - Milano, Italy.*
E-mail: anna.pandolfi@polimi.it

Keywords: Distributed fibers, Hyperelasticity, Constitutive Modeling.

Natural bio-materials represent notable examples of complex materials in which the microstructural features significantly influence the macroscopic response in a non-trivial way. An important question emerging in the definition of predictive numerical models concerns with the ability of a particular mechanical test to reveal the micro-mechanical features of the tissue that might be activated during loading. In this regards, the mechanical consequences of specific microstructural (collagen) architectures have not been investigated sufficiently in the literature, especially concerning generalized spectral methods for the constitutive modeling of the fiber reinforcement [1].

State-of-the-art of analytical and numerical tools concern the distributed characterization of the anisotropic strain energy density of soft hyperelastic materials embedded with fibers. Considering spatially distributed orientations of fibers, the analysis focuses on material models dependent on the fourth pseudo-invariant of the Cauchy-Green tensor, and on exponential forms of the fiber strain energy function. Under different loading conditions, closed-form expressions of the probability density function (PDF) of the main microstructural features of the materials represent a reliable way to implement these complex models by using a generalized random variable transformation strategy [2, 3].

As the main point of applicability, the cumbersome extension-contraction switch [4] commonly adopted for shutting down the contribution of contracted fibers in models based on generalized structure tensors can be directly tackled via the knowledge of the derived PDFs [2]. Within such stochastic framework, we propose numerical examples of simple tension, biaxial and simple shear loading patterns comparing and contrasting different values of the mean direction of collagen fibers. Our analysis supports the proposed theoretical reasoning towards robust patient-specific computational applications.

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Dynamical loading on anisotropic soft anisotropic materials and tissues

Pasquale Vena
Department of Chemistry Materials and Chemical Engineering, Politecnico di Milano, Milano, Italy
E-mail:pasquale.vena@polimi.it; dario.gastaldi@polimi.it

Keywords: Poroelasticity, Anisotropy, harmonic loading.

This work is focused on the dynamical mechanical analysis of soft hydrated anisotropic tissue as a tool for the mechanical characterisation of the biological tissues and anisotropic time dependent materials.

In previously published studies [1], the application of harmonic loading on the boundary of hydrated tissues was studied with particular reference to dynamic AFM-based tissue characterisation. The application of a sinusoidal displacement of a spherical indenter on the tissue surface, and the measure of the reaction force on the cantilever of the AFM, has been used with the purpose to identify frequency dependent elastic response of the tissue as well as its hydraulic permeability. Mechanical response of harming loading on materials and tissues can be of interest for the Magnetic Resonance Elastography (MRE) [2]. In MRE, a stationary mechanical wave is propagated within the tissue through a suitably selected mechanical actuator and the displacement field is acquired though a synchronized phase contrast magnetic resonance imaging. In both the above mentioned applications the tissue is subjected to harmonic displacement fields and the governing equation of the direct problem (wave propagation) are the same.

In this study, the effect of material/tissue anisotropy, its permeability (and anisotropic permeability) and densities of the constituents are investigated with reference to both the dynamic nanoindentation experiment and the MRE. The governing equations for the stationary wave propagation through an anisotropic fluid filled solid and their implementation into a finite element discretisation are here investigated and the specific roles played by tissue mechanical anisotropy and by material densities are studied. The results show that dynamic nanoindentation technique is able to identify the frequency dependent elastic properties of the superficial layers of the tissue; mechanical and hydraulic permeability can also be identified. Furthermore, numerical simulations of the stationary wave propagation disclose the relationship between tissue anisotropy and quantities which are measurable through MRI such as orientation of the wave propagation vector and spatial wave lengths along different directions. These finding are the basis for the implementation of inverse methods which would allow to identify mechanical properties from MRE on tissues like skeletal muscle or brain also in clinical settings.

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On a singular-value multiplicity problem arising in H_∞-norm based optimization of forced dynamic systems

Paolo Venini
Department of Civil Engineering and Architecture, University of Pavia, Pavia, Italy
E-mail: paolo.venini@unipv.it

Keywords: Singular Value Decomposition, Topology Optimization

Structural optimization of dynamic structures is being investigated by several research groups all over the world focusing either on the free vibrations of the autonomous system or on the forced response, mostly in the frequency domain. As to the former approach, a min-max problem is often times considered that aims to maximize the amplitude of the smallest eigenvalue and peculiar remedies are then found to handle pathological cases such as the appearence of multiple eigenvalues [1] or eigenvalue crossings in the design space. [2]. As for methods that account for the forced response in the frequency domain, an approach that minimizes the H_∞-norm of the system transfer function has been recently proposed in [3] that minimizes the peak-gain of Multi-Input/Multi-Output (MIMO) structures. Even in this forced case, a pathological situation shows up when two (or more) competing poles at different frequency show up that are characterized by identical (or close) values of the frequency response function. The merit function, i.e. the transfer function H_∞-norm, happens then to be continuous but not differentiable and the design oscillates back and forth between the two frequencies of the two (or more) poles. Objective of this paper is to propose a methodology that is numerically capable of handling this singular-value multiplicity ensuring a stabilization of the design procedure. The main idea is in a sense borrowed from the generalized eigenvalue defined in [2] that explicitly accounts for a few eigenvalues (and not only for the minimum one) and in fact a novel objective function is introduced that encompasses the maximum singular value, i.e. the H_∞-norm, and a few more singular values. Numerical results concerning 1D and 2D structures are eventually presented that validate the theoretical framework.

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Tailoring physical and geometric properties of ionic polymer metal composites toward optimised compressive sensing

Valentina Volpini[1], Lorenzo Bardella[1], Maurizio Porfiri[2]
[1] *Department of Civil, Environmental, Architectural Engineering and Mathematics, University of Brescia, Brescia, Italy*
E-mail: v.volpini@unibs.it, lorenzo.bardella@ing.unibs.it

[2] *Department of Mechanical and Aerospace Engineering, Tandon School of Engineering, New York University, Brooklyn, NY*
E-mail: mporfiri@nyu.edu

Keywords: ionic polymer metal composite; 3D printing; electrochemomechanics.

Ionic Polymer Metal Composites (IPMCs) are constituted by an ionomeric membrane, that is an electroactive polymer, sandwiched between two noble metals acting as electrodes in the migration of mobile ions within the membrane.

The recent IPMCs' manufacturing by 3D printing enables a customisable multilayer membrane structure in which, by acting on the ionomer density, each layer may have different properties. This opens a wide range of possibilities for multifunctional design of IPMCs as sensors and actuators.

The objective of this work is to demonstrate the potential of layer by layer deposition to enhance the performance of IPMC sensors. Specifically, we focus on IPMC compression sensing, consisting in the electrical response generated by IPMCs subject to time-varying through-the-membrane-thickness shortening displacement under short-circuited electrodes. A key feature of compression sensing is that it requires an appropriate asymmetry in the membrane's mechanical properties [1].

For such a problem, we obtain a closed-form analytical solution on the basis of the electrochemo-mechanical theory proposed in [2]. By extending the results presented in [1], here we also account for the effect of the very large variation of material properties in the thin interphase regions next to the electrodes, referred to as Composite Layers (CLs) [3]. In these regions, in which the interstitial sites within the ionomer are occupied by metal particles, the electrochemical properties sharply vary with respect to those characterising the bulk membrane. Such variations are particularly relevant because the IPMC is governed by the depletion and accumulation of mobile ions at the interfaces between bulk membrane and CLs.

By using the results previously illustrated in [3], where the mechanical contribution to the IPMC response is neglected, we formulate an equivalent circuit model, solved through the method of matched asymptotic expansions. This semi-analytical solution, validated against finite element simulations, is then employed to perform parametric analyses, leading to the determination of a possible optimal layered structure for IPMC applications.

This study is expected to lay the foundation for future endeavors on the optimisation of the mechanical properties of 3D printed IPMCs in other important domains.

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Enhanced micromorphic modelling of beam lattices: a 1D introductory approach

Andrea Bacigalupo[1], Luigi Gambarotta[2]
[1]*IMT School for Advanced Studies, Lucca, Italy*
E-mail: *andrea.bacigalupo@imtlucca.it*

[2] *Department of Civil, Chemical and Environmental Engineering, University of Genoa, Genoa, Italy*
E-mail: *Luigi.gambarotta@unige.it*

Keywords: periodic beam lattices, enhanced micromorphic models, dispersive waves.

The homogenization of beam lattices composed of slender ligaments connecting nodes having translational and rotational inertia presents some controversial points. In fact, the homogenization of the Lagrangian model with translation and rotation of the nodes it may be carried out to obtain a micropolar continuum according two different procedures [1]. In the first case a second order expansion of the generalized displacement field is assumed as a down-scaling law to be substituted in the equation of motion of the Lagrangian system. It has been shown that the resulting second order elasticity tensor relating micro-curvatures to micro-couples turns out to be negative defined. However, when considering the harmonic Bloch wave propagation, this model provides an accurate simulation of both the acoustic and the optical branches of the Lagrangian model. On the other side, the second approach, that is based on the generalized macro-homogeneity condition assuming a down-scaling law with the rotational field truncated to the first order, provides a positive defined second order elasticity tensor. However, it has been shown that this model fails in the simulation of the optical branch of the Lagrangian lattice [1].
This drawback may be attributed to a limited accuracy of the down-scaling law and of the description of the inertial effects in the homogenized continuum. An attempt is here carried out to obtain an 1D enhanced continualization procedure that seems to be able to solve other well-known drawbacks in the homogenization of mono-dimensional lattices. The generalized displacement field at the nodes close to the reference one is expressed through the shift operators and the down-scaling law is obtained by the concept of central difference expressed in terms of such operator, by generalising a method proposed by Rosenau [2,3]. By a proper expansion of the resulting governing equation in terms of differential operators, a new generalized micromorphic continuum is obtained that is also characterized by non-local inertia terms. The resulting continuum turns out to be characterized by a positive defined elastic energy density. Several lattice systems are analized with a good agreement between the static and the dynamic response between the discrete and the continuum model. These results may support further developments of the method to describe 2D and 3D beam lattice models.

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A FE-BIE coupled method for the static analysis of beams on 3D half-space

Daniele Baraldi[1], Nerio Tullini[2]
[1]*Department of Architecture, Construction, Conservation, Università IUAV di Venezia, Venice, Italy*
E-mail: *danielebaraldi@iuav.it*

[2]*Department of Engineering, University of Ferrara, Ferrara, Italy*
E-mail: *nerio.tullini@unife.It*

Keywords: Soil-structure-interaction, mixed variational principle, Boussinesq solution

This contribution presents a simple and effective numerical model for performing static analyses of beams in frictionless and bilateral contact with a three-dimensional (3D) elastic and isotropic half-space. Such a problem can suitably represent the behaviour of strip footings and shallow foundations in building structures.

A coupled finite element-boundary integral equation (FE-BIE) method, already introduced for beams on half-plane [1], is here extended to the case of beams on 3D half-space, by adopting a mixed variational formulation which assumes as independent fields both beam displacements, rotations and contact surface tractions. Mixed formulation includes the Green function of the 3D half-space [2]. The resulting numerical model makes use of locking-free "modified" Hermitian shape functions [3] for the beam and piecewise constant function for the substrate tractions. For this purpose, only the contact surface underneath the foundation needs to be discretized with rectangular elements; furthermore, traction distribution in beam transverse direction is considered with an adequate mesh refinement of contact surface in both plane directions.

Numerical tests of both Euler-Bernoulli and Timoshenko beams subject to several load conditions are dealt with. Results in terms of displacements, contact tractions and bending moment are obtained, showing the effectiveness of the model and its convergence to existing analytical and numerical solutions [4].

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Three-leaf brick masonry walls: FE & DE models

Daniele Baraldi[1], Claudia Brito de Carvalho Bello[1], Antonella Cecchi[1], Emilio Meroi[1], Emanuele Reccia[2]
[1]*Department of Architecture Construction Conservation (DACC), Università IUAV di Venezia, Venice, Italy*
E-mail: *danielebaraldi@iuav.it, decarvalho@iuav.it, cecchi@iuav.it, meroi@iuav.it*

[2]*Department of Structural and Geotechnical Engineering (DISG), Sapienza University of Rome, Rome, Italy*
E-mail: *emanuele.reccia@uniroma1.it*

Keywords: Three-leaf masonry walls, interface stiffness, numerical simulation

Most Europeans historical buildings are made by brick or stone masonry, often constituted by multiple leaves. A widespread multi-leaf typology is the three-leaf brick masonry wall, which presents two external brick leaves and one inner core consisting of different incoherent materials, with large presence of voids. Brick masonry behaviour tends to brittle failure mechanisms. By reference to solid masonry, multiple leaf walls present a more complex mechanical behaviour, with not simple, but combined collapse mechanisms: their interpretation remains a challenge [1-3]. The literature referred to modelling masonry interface behaviour considers mortar joints reduced to interfaces and internal block interfaces [4]. Here attention is focused on modelling interface between masonry layers: contact relationships between the inner core and the external leaves. This work develops interface models to describe the internal load distribution in a multi-leaf masonry wall, in addition to existing models already adopted for one-leaf masonry elements [5]. Compressive load results are evaluated with reference to experimental data recently obtained by the research group [6] by means of tests carried out at Laboratorio di Scienza delle Costruzioni IUAV (Labsco). This investigation aims to develop a suitable approach to predict the performance of historical masonry wall by FE & DE simulations. Here the two numerical modelling strategies on three-leaf brick masonry walls are presented and compared.

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An eXtended Virtual Element Method for the Laplace problem with strong discontinuities and vertex singularities

Elena Benvenuti[1], Andrea Chiozzi[1], Gianmarco Manzini[2], N. Sukumar[3]
[1]*Department of Engineering, University of Ferrara, Ferrara, Italy.*
[2]*T-5 Group, Los Alamos National Laboratory, Los Alamos, New Mexico, USA.*
[3]*Department of Civil & Environmental Engineering, University of California, Davis, USA.*
E-mail: *elena.benvenuti@unife.it, andrea.chiozzi@unife.it, gmanzini@lanl.gov, nsukumar@ucdavis.edu.*

Keywords: virtual element method; extended finite element method; singularities.

The Virtual Element Method (VEM) [1] directly emanates from the Mimetic Finite Difference Method and can be regarded as a generalization of the Finite Element Method, that is capable of dealing with very general polygonal or polytopal meshes. The fundamental feature of VEM is the introduction of a suitable projector operator which approximates the bilinear form characterizing the weak formulation of the continuous problem, so that the explicit construction of the elemental basis functions can be avoided. The VEM has recently aroused significant interest also in structural mechanics, where it has been applied, among others, to linear and finite elasticity problems [2-3]. In particular, polygonal and polytopal meshes can be especially suited to complex domains, such as those occurring in bodies with cracks or re-entrant corners. Unfortunately, such problems involve additional difficulties due to the existence of discontinuities and singularities in the unknown fields. In the last decades, this issue has been successfully addressed by the eXtended Finite Element Method (XFEM) [4,5], which relies on the enrichment the approximation space by means of additional shape functions specifically tailored to reproduce the non-smooth features of the expected solution.

This contribution aims to investigate the capability of VEMs to approximate solutions with jump discontinuities and vertex singularities for the Laplace problem. This goal is achieved by enriching the virtual element function space with suitable additional basis functions. The advantages of the proposed eXtended VEM will be discussed through representative numerical examples.

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Multiple strain localization and debonding failure mechanisms of thin quasi-brittle coatings

Guido Borino[1], Francesco Parrinello[1]
[1]Department of Civil, Environmental, Aerospace, Materials, University of Palermo, Palermo, Italy.
E-mail: *guido.borino@unipa.it, francesco.parrinello@unipa.it*

Keywords: strain localization, interface debonding, nonlocal damage.

Thin ceramic films are often superposed to hard superalloys mechanical systems, with the double purpose of protection from oxidation attacks, and to be an effective thermal barrier against high temperature variations [1].
The mechanical system is then represented by a substrate, with high elastic and strength properties with one, or more than one, coating thin films possessing high thermal insulation properties, but with a quasi-brittle behaviour, induced by the constitutive nature of the ceramic material.
A further weakness is represented by the adhesive interface between the substrate alloy and the coating film (or also between films, if more than one coating layer is adopted).
The paper investigates, from a computational point of view, the mechanical response of a two dimensional element with a surface coating subjected to thermal and mechanical loadings.
Two main mechanical failure mechanism are observed, namely: a) the development of vertical cracks in the coating, due to tensile stresses and to the quasi-brittle constitutive nature of the coating; b) the decohesion and progressive debonding along the interface between the substrate and the film.
The first mechanism is reproduced adopting a nonlocal damage model [2], which allows to obtain a finite element objective solution showing multiple vertical cracks located at specific distance each other. The latter multiple strain localization is then followed by the formation and development of the delamination, which is modelled by a cohesive-frictional interface elements [3]. The final failure mechanisms are then reported, which shows the expulsion of small parts of the coating.
The main characteristic of the mechanic and thermal responsees are finally discussed and some conclusion related to the optimal coating thickness is addressed.

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Snap-back mechanisms of planar rods subject to kinematical boundary conditions

Alessandro Cazzolli[1], Francesco Dal Corso[1], Davide Bigoni[1]
[1] *DICAM, University of Trento, Trento, Italy*
E-mail:a.cazzoll.1@unitn.it,francesco.dalcorso@unitn.it,davide.bigoni@unitn.it

Keywords: elastica, snap-back, instability

The aim of the present research is to disclose the whole set of stable quasi-static configurations for an inextensible and planar elastic rod kinematically constrained at its ends by means of moving and rotating clamps. The stable equilibrium configurations are described for every possible combination in the kinematical boundary conditions, that can be reduced to the two imposed rotations at the ends and the distance between the two clamps. In order to predict the occurrence of snap-back mechanisms during a generic evolution in the boundary conditions, the complete set of critical kinematical parameters is defined as corresponding to the transition from stable to unstable equilibrium configurations. Such critical set is represented within the space having for axes the corresponding values of the kinematical parameters, revealing the presence of two universal critical surfaces for the considered structural system. In particular, cross sections of these surfaces show the typical shape of catastrophic cusps, revealing the strong influence of small imperfections in the boundary conditions' incremental regimes (with respect to the purely symmetric and antisymmetric cases) in terms of critical snap-back angles.

The differential equation, obtained through a variational method and governing the equilibrium configurations of the structural system, is integrated by means of the Jacobi elliptic functions into a very general formulation. The stability of the equilibrium configurations is analysed solving the related Sturm-Liouville isoperimetrically constrained problem, which governs the eigenfunctions related to the perturbation field.

The analytical predictions are confirmed by numerical simulations performed by means of the commercial code Abaqus. Results from present model are fully validated by experimental data available for symmetric boundary conditions [1], [2] as well as for non-symmetric configurations by own experimental activity performed on a physical model.

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Damage and Plasticity Model for Strain Gradient Materials and numerical applications

Massimo Cuomo[1], Loredana Contrafatto[1], Leopoldo Greco[1]
[1] *Dipartimento di Ingegneria Civile ed Architettura (DICAR),*
Universitá degli studi di Catania, Catania, Italy
E-mail:mcuomo@dica.unict.it, lcontra@dica.unict.it, leopoldo.greco@virgilio.it

Keywords: Materials microstructure, damage, non-local plastic model.

For modern architected materials microstructure has a strong influence on the mechanical behaviour, either in the elastic range and in the anelastic range. Independent mechanisms for the evolution of the anelastic phenomena at the micro- and at the macro-scale can be recognized, giving rise to non-local plastic models, usually fairly more general than the classical gradient plasticity model, originally introduced for geomaterials. Also continuum damage has been extended to micromorphic and higher gradient models, introducing internal damage variables and their gradient [1]. In these models the damage evolution is determined by rate equations depending upon the thermodynamic conjugate forces, related to the local value of the damage variables and to its gradient. An additional field differential equation is then obtained for damage compatibility, that has to be solved simultaneously with the equilibrium equations.

In this work is proposed a new continuum damage model for strain gradient materials characterized by two independent damage internal variables, each one characterised by an evolution law, that can be solved locally. In this way no additional field equation for the damage variable is needed. Non-locality is then accounted in the equilibrium and compatibility equations. The derivation has been obtained within a thermodynamic framework, introducing suitable functionals for the internal energy and for the rate of dissipation.

Specific objectives of the work are:
- to present a damage model for strain gradient materials that does not need the solution of additional differential equations;
- to analyze some simple version of the model coupled with plasticity through 2D numerical simulation obtained using isogeometric interpolation.

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Canonical quasicrystalline waveguides

Massimiliano Gei[1], Lorenzo Morini[1], Zhijiang Chen[1]
[1]*School of Engineering, Cardiff University, Cardiff CF24 3AA, UK*
E-mail: *geim@cardiff.ac.uk, morinil@cardiff.ac.uk*

Keywords: metamaterial, phononic crystal, Fibonacci structure.

The Floquet-Bloch spectrum of a family of one-dimensional two-phase periodic structured rods generated by a quasicrystalline sequence -such as the Fibonacci recursion rule- is characterised by self-similar stop/pass band layouts and scaling phenomena [1,2]. These properties are governed by an invariant function of the circular frequency, the Kohmoto's invariant, which can be represented as a surface in a suitable three-dimensional space. In general, the traces of the transmission matrices of the family of rods identify orbits on the Kohmoto's surface [2]. For particular frequencies, named *canonical* frequencies, the orbits are closed. The condition for the existence of these frequencies is that a particular ratio between the constitutive parameters of the two phases composing the waveguide is a rational number. We show that for these canonical structures the scaling of the spectrum can be determined analytically performing a linearisation of the orbit concerned. We also show that for frequencies that are not canonical, there are two types of stop bands for the same rod, regular and ultrawide. For both, an analytical estimation is provided.

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Dynamic modelling of a gyroscopic stabilizer for the flutter performances of a long span bridge deck

Gian Felice Giaccu[1], Antonio Cazzani[2]
[1]*Department of Architecture, Design and Urban Planning (DADU), University of Sassari, Alghero, Italy*
E-mail: *gf.giaccu@uniss.it,*

[2]*Department of Civil, Environmental Engineering, and Architecture (DICAAR), University of Cagliari, Cagliari, Italy*
E-mail:*antonio.cazzani@unica.it*

Keywords: Gyroscopic stabilizer, long span bridges, flutter performances.

Long span suspended bridges are challenging structures which are generally sensitive to wind effects and to wind-induced instabilities: it is well known that torsional rotation of the bridge deck plays an important role for the dynamic stability of the structure [1].

The paper focuses the attention on a gyroscopic device conceived as an active stabilizer of the long span bridge: the proposed method has been applied to a 3 degrees-of-freedom (dofs) section-model of the bridge deck.

A mathematical model of the gyroscopic device taken form [2] has been adopted by the authors in the present paper; the main component of the system is a rotating mass, with a horizontal angular momentum **Ω,** which is parallel to the section-model; the mass is connected to the deck by a rotational spring. The system allows relative rotations between the section-model and the rotating mass in the bridge deck plane. The gyroscopic system reacts to torsional vibrations of the deck by coupling torsional vibrations with the other dofs of the system, thus transferring the energy from the torsional mode to the others frequencies of the system [3, 4].

Free vibrations of the section-model were analyzed in order to assess the effectiveness of the proposed apparatus.

Results show that the use of a gyroscopic stabilizer allows, as expected, for an effective reduction of torsional rotation amplitude of the section-model.

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Meta-heuristic algorithms for the kinematic NURBS-based limit analysis of curved masonry structures

Nicola Grillanda[1], Andrea Chiozzi[1], Gabriele Milani[2], Antonio Tralli[1]
[1]*Department of Engineering, University of Ferrara, Ferrara, Italy*
E-mail: *andrea.chiozzi@unife.it, nicola.grillanda@unife.it, tra@unife.it*

[2] *Department of Architecture, Built Environment and Construction Engineering (A.B.C.), Technical University of Milan, Milan, Italy*
E-mail: *gabriele.milani@polimi.it*

Keywords: masonry, NURBS, meta-heuristic algorithm.

The modern theory of limit analysis has proven to be one of the most reliable methods for assessing the ultimate load bearing capacity of masonry structures since the strong heterogeneity of the mechanical properties makes linear elastic approaches unsuitable.
The recently developed computational methods for masonry vaults can be classified into two broad categories: the Finite Element methods developed both for nonlinear incremental analysis and for limit analysis, and the thrust network methods, directly based on a lower bound formulation.
In the literature, many kinematic (i.e. upper bound) approaches have been proposed for the analysis of curved masonry structures (i.e. arches and vaults). However, such procedures are usually highly inefficient. Recently, the Authors proposed a new Genetic Algorithm NURBS-based approach for the homogenized kinematic limit analysis of masonry vaults based on an upper bound formulation in which a NURBS description of the vault geometry is used to generate a very coarse mesh of the vaulted surface in which each element is idealized as a rigid body [1-2]. Then, an upper bound limit analysis problem with very few optimization variables can be devised, which takes into account the main aspects of masonry material through homogenization, where dissipation is allowed along element edges only. Due to the very limited number of rigid elements used, the collapse load so found depends on the position of element interfaces. Mesh adjustments are therefore needed and can be performed by employing simple meta-heuristic approaches. In this contribution, different meta-heuristic mesh-adjustment algorithms are compared and critically discussed. In particular, the Genetic Algorithm (GA) proposed in the original papers is here compared with the Particle Swarm Optimization (PSO) Algorithm [3], the Firefly Algorithm (FA) [4] and an improved version of the Prey Predator Algorithm (PPA) [5]. The improved PPA proves to be the most efficient mesh-adjustment approach in the proposed NURBS-based limit analysis procedure.

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Quasi-mosaic crystals and their applications in modern research

Vincenzo Guidi[1]
[1]*Department of Physics and Earth Sciences, Ferrara, Italy*
e-mail: *guidi@fe.infn.it*

Keywords: quasi-mosaicity, anisotropic mechanical properties

Quasi-mosaicity is an effect of anisotropy in crystal deformation, which permits the generation of two curvatures in two perpendicular families of crystallographic planes [1]. Important applications in modern science are already operational or envisaged by this effect, such as steering of charged-particle high-energy particle beams or focalization of hard X- and soft gamma-rays. About the first, successful achievements have been reached at many accelerators, and particularly at the Large Hadron Collider (CERN) Geneva, Switzerland, with highest world-wide energy of 7 TeV protons. For the second application, a prototype of a Laue lens has been built with quasi-mosaic crystals with the aim of focusing an x-ray beam for oncologic treatment, operating with much softer photons than in conventional radiotherapy—a technique which may result in a simpler irradiation scheme and with better space resolution. A technique to obtain self-standing curved crystals has been developed. The method is based on sandblasting, which is capable of producing an amorphized layer on the crystals. It has been demonstrated that the amorphized layer behaves as a thin compressive film, causing a curvature in the substrate. This procedure permits the fabrication of homogeneously curved crystals in a fast and cheap way. Several samples were manufactured and bent using the sandblasting method at the Sensor and Semiconductor Laboratory of Ferrara, Italy. Their curvature was verified using interferometric profilometry, exhibiting a deformation in agreement with the Stoney formalism. The curvature of the machined samples was also tested using gamma-ray diffraction at the Institut Laue–Langevin and at the European Synchrotron Radiation Facility at Grenoble, France. A good agreement with the dynamical theory of diffraction was observed. In particular, the experiment showed that the crystalline quality of the bulk was preserved. Moreover, the method allowed curved samples to be obtained free of any additional material. Finally, a crystalline undulator was produced using sandblasting and tested using gamma-ray diffraction at the ILL.
A comprehensive survey of the theory of quasi-mosaicity based on linear theory of elasticity in an anisotropic medium will be worked out in this report as well as an illustration of the methods to attain it. Special focus to current applications with also be given.

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An isogeometric integral continuum damage approach

Vincenzo Mallardo[1], Eugenio Ruocco[2], Gernot Beer[3]
[1]*Department of Architecture, University of Ferrara, Ferrara, Italy*
E-mail: *mlv@unife.it. Corresponding author*

[2]*Department of Civil Engineering, Design, Building and Environment, University of Campania, Aversa, Caserta, Italy*
E-mail: *eugenio.ruocco@uninacampania.it*

[3]*Institute of Structural Analysis, Graz University of Technology, Graz, Austria*
E-mail: *gernot.beer@tugraz.at*

Keywords: BEM, Three-dimensional (3D).

The isogeometric analysis has opened new perspectives on the application of the classical Boundary Element Method (BEM) to the nonlinear structural analysis. In such a context most of the discretization concerns the boundary surfaces, whereas a numerical domain representation needs to be adopted for the zone involved by the nonlinear process.

Based on recent works of the authors [1-2], in the present contribution an isogeometric boundary element method (IGABEM) based on non-uniform rational B-splines (NURBS) approach is presented in three-dimensional continuum damage analysis. The domain can contain subvolumes that exhibit local damage.

The description of the geometry of the problem and the governing field values are approximated by different NURBS basis functions. The geometry of the damaged zone is represented by the aid of two NURBS surfaces and a special mapping method.

The advantage over currently used methods is that no discretization into cells is required to evaluate the arising volume integrals and that the geometry independent approximation allows flexible refinement options. The implementation is verified on test cases.

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Effective properties of composites containing toroidal inhomogeneities

Enrico Radi[1], Igor Sevostianov[2] Luca Lanzoni[3],
[1]*DISMI, Università di Modena e Reggio Emilia, 42122 Reggio Emilia, Italy*
E-mail: *enrico.radi@unimore.it*

[2]*DMAE, New Mexico State University, Las Cruces, USA*
E-mail: *igor@nmsu.edu*

[3]*DIEF, Università di Modena e Reggio Emilia, 41125 Modena, Italy*
E-mail: *luca.lanzoni@unimore.it*

Keywords: composites; toroidal inclusion; effective properties.

The present work focuses on the problem of a rigid inhomogeneity of toroidal shape embedded in an elastic matrix. Inhomogeneities of this kind occur in both natural and man-made materials. Barium titanate nanotori are used as nonvolatile memory devices, transducers, optical modulators, sensors and possible energy storage in supercapacitors. Toroidal particles represent preferred morphology of Li2O2 deposition on porous carbon electrode in lithium-oxygen batteries. Polymeric "microdonuts" are used in bioengineering; toroidal shape of nanoparticles is preferred for microwave absorption properties of $BaTiO_3$. Toroidal particles of SiO_2 may form in a Cu matrix due to internal oxidation of a Cu-Si solid-solution polycrystal. Analytical modeling of materials with such microstructure has not been well developed. In the homogenization schemes, the inhomogeneities are usually assumed to be of ellipsoidal shape. This unrealistic assumption is responsible for insufficient linkage between micromechanics and materials science applications.

While for 2D non-elliptical inhomogeneities many analytical and numerical results have been obtained, only a limited number of approximate estimates are available for non-ellipsoidal 3D shapes. Asymptotic methods have been used in [1] to evaluate the contribution of a thin rigid toroidal inhomogeneity into overall stiffness. Eshelby tensor for a toroidal inclusion has been also derived by Onata. However, Eshelby tensor for non-ellipsoidal inhomogeneities is irrelevant to the problem of effective properties of a heterogeneous material. The effective conductivity of a material containing toroidal insulating inhomogeneities has been addressed in [2].

We first consider a homogeneous elastic material, with isotropic stiffness tensor $\mathbb{C}^0$, containing a rigid inhomogeneity of volume $V^{(1)}$. The contribution of the inhomogeneity to the overall stress per representative volume V (the extra stress $\Delta\boldsymbol{\sigma}$, as compared to the homogeneous matrix) is given by the fourth-rank stiffness contribution tensor $\mathbb{N}$, defined by the following relation

$$\Delta\boldsymbol{\sigma} = \frac{V^{(1)}}{V}\,\mathbb{N}\,\boldsymbol{\varepsilon}^{\infty} = \frac{1}{V}\int_S \boldsymbol{\sigma}\,\boldsymbol{n}\otimes\boldsymbol{x}\,dA \tag{1}$$

where $\boldsymbol{\varepsilon}^{\infty}$ is the remotely applied strain, $\boldsymbol{n}$ is the outward unit normal to the inhomogeneity surface S. To calculate the components of $\mathbb{N}$, a displacement boundary value problem has been solved for 3D elastic space containing a rigid toroidal inhomogeneity.

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Experimental study on mechanical properties of NFRCM using various matrices and validation by X-Ray micro tomography.

Renato S. Olivito[1], Rosamaria Codispoti[1], Carmelo Scuro[1], Raffaele Giuseppe Agostino[2], Raffaele Filosa[2], Vincenzo Formoso[2]

[1]*Department of Civil Engineering,*
University of Calabria, Rende, Italy
E-mail: {renato.olivito; rosamaria.codispoti; carmelo.scuro}@unical.it

[2]*D STAR Lab and Department of Physics,*
University of Calabria, Rende, Italy
E-mail: {raffaele.agostino; raffaele.filosa; vincenzo.formosa}@unical.it

Keywords: NFRCM, structural strengthening, X-Ray micro tomography.

The idea of alternative substitution to common composite materials (FRP) with sustainable composite materials is rapidly developing. The CO2 emissions during the construction phase of a building, but also during the use of them, are equal to approximately 40% of all the CO2 emissions; 15% of which are related to the energy consumption during the production of construction materials [1]. However, the sustainable composite materials, indicated with the acronym NFRCM (Natural Fibers Reinforced Cementitious Matrix), are eco-friendly both in the reinforcing fibers that in the matrix, and to date do not exist technical standards which set out their design rules or composition. To analyse the mechanical behaviour of NFRCM, standards regarding FRP or common FRCM (Fibers Reinforced Cementitious Matrix) materials are taken into account. It is however necessary to make changes regarding the production stages of NFRCM materials respect to the common composite materials [2-3]. For the purpose of this work, a type of natural fibers made with basalt was examined. This type of fibers have been studied in previous works [3] carrying out tensile tests. Furthermore, various matrices with different granulometric composition and mechanical properties were considered during the manufacturing of composite materials in the laboratory. Specifically, two organic and one inorganic matrices were investigated. In the phase before preparation of NFRCM specimens, as the fabrics, also the matrices have been analysed from experimental point of view carrying out compressive tests and three points bending tests. From the obtained results, the performance of used matrices was classified and analysed in terms of compression strength, bending strength and tensile strength.

An x-ray micro tomography has been carried out in order to analyse the adhesion status between the cementitious matrix and the basalt fibers. The in-line phase-contrast x-ray imaging and tomography have used to obtain a non-destructive 2D and 3D characterization of the specimens and to detect cracks and voids. A complete set of 2D projections is acquired and a 3D map of the X-ray absorption in the volume is mathematically reconstructed [4]. The 3D map permitted to identified the precise position of the voids and damage in the volume as well as to determine the components distribution in the sampled volume.

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Cross section reduction upon constant intrados stress in curved beams

Antonio Strozzi[1], Enrico Bertocchi[1], Sara Mantovani[1]
[1]*Department f Engineering 'Enzo Ferrari', University of Modena and Reggio Emilia, Modena, Italy*
Department, Affiliation, City, Country
E-mail: *astrozzi@unimore.it, ebertocchi@unimore.it, sara.mantovani@unimore.it*

Keywords: curved beam, paradox, cross section reduction.

A paradoxical behaviour is known to exist for a straight beam subjected to bending, according to which it is sometimes possible to lower the bending stress by removing material from the zones farthest from the section neutral axis. A rhombic section from which the top and bottom edges are removed is a convincing example, [1]. Moving to a curved beam, it has recently been shown that it is similarly possible to lower the bending stress by laterally removing material from regions close to the neutral axis of the beam section. Again, the rhombic section constitutes an instructive example, where this time the lateral edges are removed. This mechanical response too may be classified as paradoxical, since it is possible to simultaneously reduce the bending stress and the beam section area. However, various numerical tests have shown that the stress diminution achievable with this approach is often of the order of 1 per cent, too small to be technically significant, whereas the section area diminution is practically more interesting, often in the region of 10 per cent. To get a more appreciable area reduction, it was decided to impose a less stringent condition, i.e. the intrados bending stress remains unvaried upon material removal. To be of help for the designer, this approach should define which beam cross section zones may be removed without producing an intrados stress variation. This approach employs the classical expression for the bending stress in a curved beam, [2]. The equation supplying the intrados bending stress is then linearized according to Frechet differentiation; then, an integration is performed for a specific shape of the lateral removal, and the condition that the material diminution does not vary the intrados stress is imposed. If this condition is coupled to the requirement that the material removal be maximum, this approach defines a horizontal strip that delimits the beam cross section zone from which material may be removed without varying the intrados bending stress. The definition of this favourable zone requires the solution of a lengthy transcendental equation, that may be achieved by adopting an iterative perturbation method. Preliminary tests have been carried out on a square cross section along whose sides two rectangular grooves are machined. It has been found that the achievable material diminution increases to 15 per cent of the initial cross section, and, therefore, it appears to be technically interesting, [3].

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Numerical analyses of a URM chimney damaged by the 2012 Emilia (Italy) earthquake

Antonio Tralli[1], Gabriele Milani[2], Fabio Minghini[1]
[1]Engineering Department, University of Ferrara, Ferrara, Italy
E-mail: *antonio.tralli@unife.it, fabio.minghini@unife.it*

[2]Department of Architecture, Built Environment & Construction Engineering
E-mail: *milani@stru.polimi.it*

Keywords: masonry chimney, modal pushover, response history analysis.

The analysis of a brickwork chimney damaged by the 2012 Emilia earthquake is presented. A survey campaign revealed evident diagonal cracks and a significant mortar deterioration in the upper part of the structure. Laboratory tests were carried out to estimate the masonry properties. Lateral force method and pushover analyses using traditional force profiles were shown to be unable to capture the seismic behavior of the chimney. In contrast, the Modal Pushover Analysis (MPA) using force distributions based on six modes of vibration led to crack patterns consistent with the observed damages. The behavior factor to be used in modal response spectrum analyses, estimated from the MPA results, was shown to be slightly higher than unity, indicating a very low dissipative capacity of the structure. Finally, a number of nonlinear Response History Analyses (RHA) were carried out using natural accelerograms. In particular, an accurate finite-element model was built using rigid triangular elements connected by means of nonlinear interfaces that incorporate suitable tension, compression, and shear constitutive laws. The results systematically emphasized a shear failure mechanism of the upper part of the chimney. The residual deformations obtained from the analyses were close to those measured after the earthquake.

After the earthquake, the upper damaged 12.40 m were disassembled for security reasons. Both before and after shortening, the ratio between the effective mass of the fundamental mode and the total mass is approximately 20%, leading standard pushover analysis methods not to be appropriate for estimating the seismic demand. For the shortened chimney, the MPA revealed damages in the lower part of the stack (8–21 m), because of a prevailing influence of the fundamental mode. In the RHA, a more evident contribution of the higher modes was observed, probably because of the effect of the vertical component of the ground motion, not accounted for in the MPA.

The main findings of this study were reported in [1, 2].

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Solution of XFEM ill-conditioned systems by penalty stabilization and a novel iterative procedure

Giulio Ventura[1], Nicola Viale[1]
[1]*Department of Structural, Geotechnical and Building Engineering, Politecnico di Torino, Torino, Italy*
E-mail: *giulio.ventura@polito.it, nicola.viale@polito.it*

Keywords: extended finite element method, stabilization, ill-conditioning.

The eXtended Finite Element Method and in general Partition of Unity Methods may show ill-conditioning of the global system of equations at solution due to near linear dependency between element shape functions and enrichment functions. This is easily observed when a discontinuity crosses nearby the element nodes and the Heaviside function enrichment is present.
In [1] it has been shown that ill-conditioning has, as limiting case, the indeterminacy of the global system of equations. This originated the idea of improving system conditioning by biasing to zero a proper subset of the enrichment variables by a weak penalty term. This technique can also be seen as a particular case of Tikhonov regularization [2], that is a commonly used method of regularization of ill-posed problems in statistics and inverse problems. This stabilization technique has no impact on the XFEM formulation, can be used for any set of enrichment functions and is of straightforward implementation.
The introduction of the stabilization terms introduces a bias in the solution, often acceptable in most applications. However, it is desirable that the solution is not affected by the stabilization procedure. Recently, an efficent way of eliminating the stabilization bias from the solution has been found by developing a new iteration formula that is an improvement of the method proposed by Barzilai and Borwein [3], who suggested formulas for the stepsize determination in steepest descent methods. The new iteration formula shows very good performance and leads to convergence in a few iterations. The derivation of the new iteration formula and examples of its application to the stabilization of XFEM problems are shown in the present work.

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Design and experimental validation of an auxetic phononic crystal for industrial micro-systems

Valentina Zega[1], Alessandro Nastro[2], Luca D'Alessandro[1], Marco Ferrari[2], Raffaele Ardito[1], Carlo Valzasina[3], Vittorio Ferrari[2], Alberto Corigliano[1]
[1]*Civil and Environmental Engineering, Politecnico di Milano, Milano, Italy*
E-mail: *valentina.zega@polimi.it, luca.dalessandro@polimi.it, raffaele.ardito@polimi.it, alberto.corigliano@polimi.it*

[2]*Department of Information Engineering, University of Brescia, Brescia, Italy*
Email: a.nastro002@unibs.it, marco.ferrari@unibs.it, vittorio.ferrari@unibs.it

[3]*STMicroelectronics, AMG R&D, Cornaredo, Italy*
E-mail: *carlo.valzasina@st.com*

Keywords: MEMS, phononic crystal, auxetic response

The design and the combination of innovative metamaterials [1] are attracting increasing interest in the scientific community because of their unique properties that go beyond the ones of natural materials. In particular, auxetic materials [2] and phononic crystals [3] are widely studied for their negative Poisson's ratio and their bandgap opening properties, respectively.
In [4] the authors combined the auxetic and phononic properties at the macro-scale to obtain a 3D structure with a wide and tunable three-dimensional bandgap.
In this work, auxeticity and phononic crystals bandgap properties are instead combined to obtain the first single material periodic micro-structure with a tunable bandgap.
A proper mechanical design, compatible with standard Micro-Electro-Mechanical Systems (MEMS) fabrication processes, of the periodic structure is here proposed and the main auxetic and bandgap properties of the innovative structure are studied through numerical simulations with COMSOLMultiphysics and analytical models.
The proposed structure consists of 3x5 auxetic unit cells rigidly connected to four external frames: top and bottom frames containing the electrodes employed for the transmission spectrum analysis, are not directly anchored to the substrate to avoid waves dispersion on the substrate, while left and right frames, properly suspended through folded springs, contain the electrodes employed for the auxetic tuning.
A prototype of the MEMS auxetic phononic crystal has been fabricated through the Thelma micro-fabrication process developed by STMicroelectronics and preliminary experimental results both in statics and dynamics prove the auxetic behavior of the structure.

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